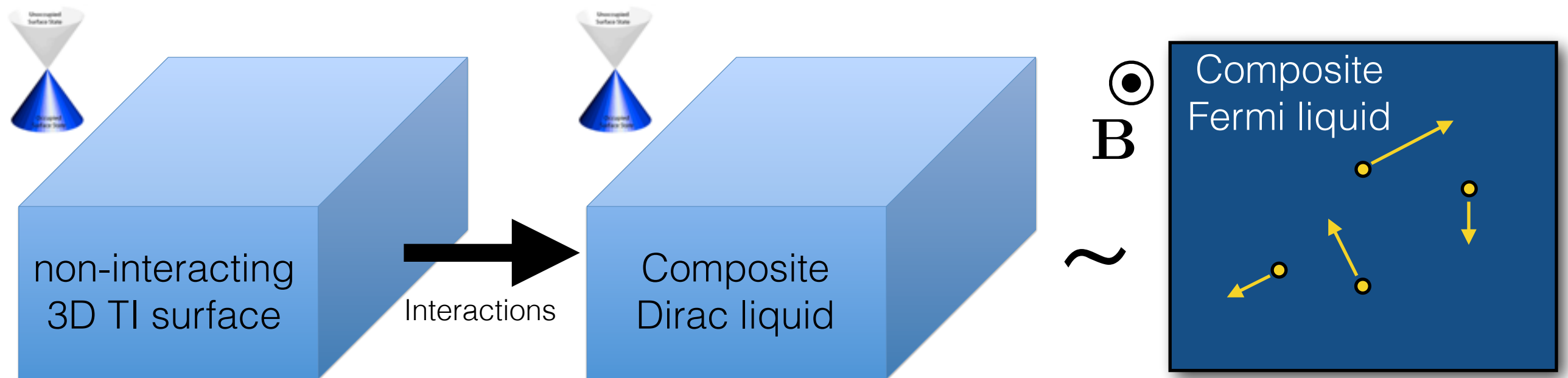


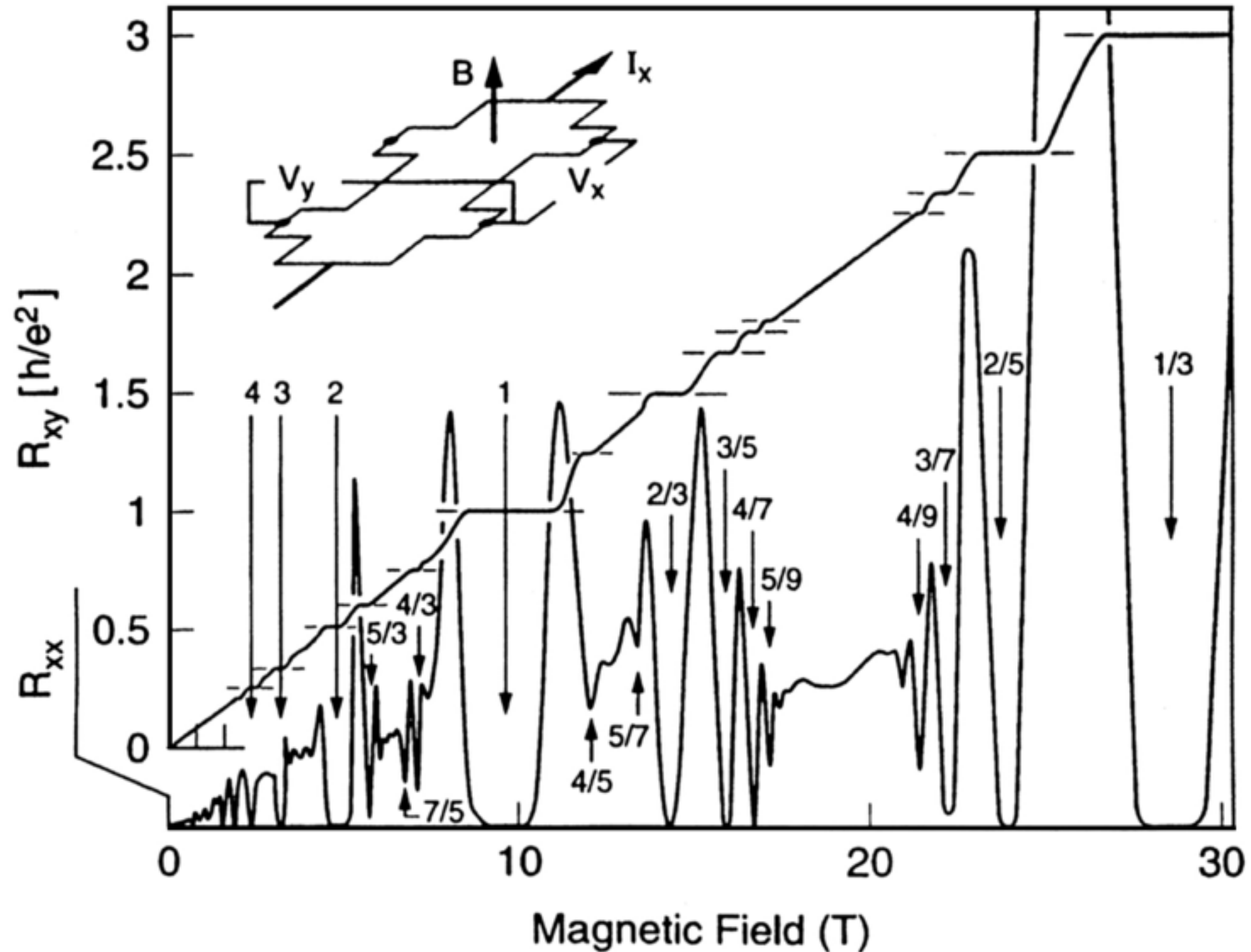
# Composite Dirac liquids



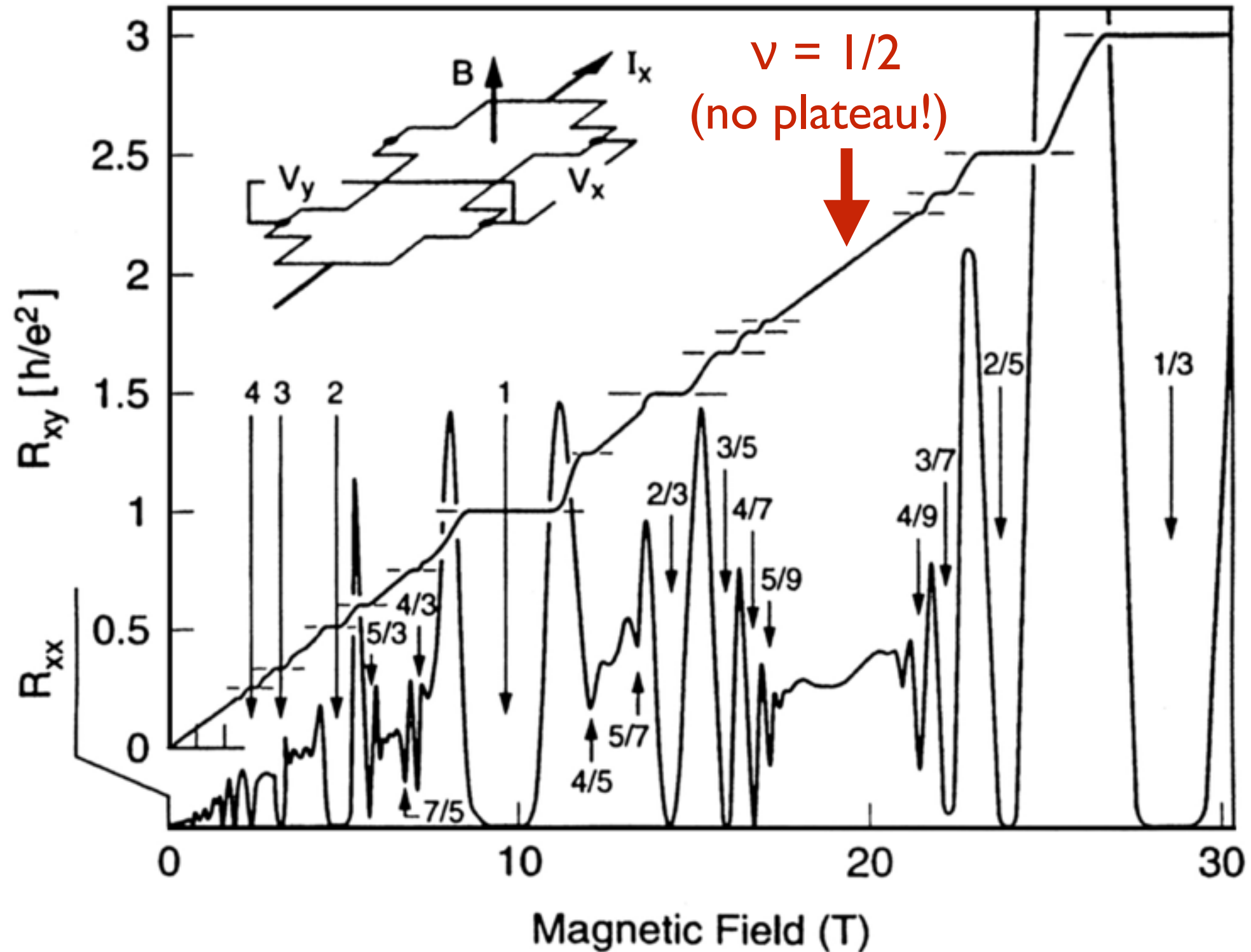
**Jason Alicea, Caltech**

David Mross, Andrew Essin, & JA,  
Physical Review X 5, 011011 (2015)



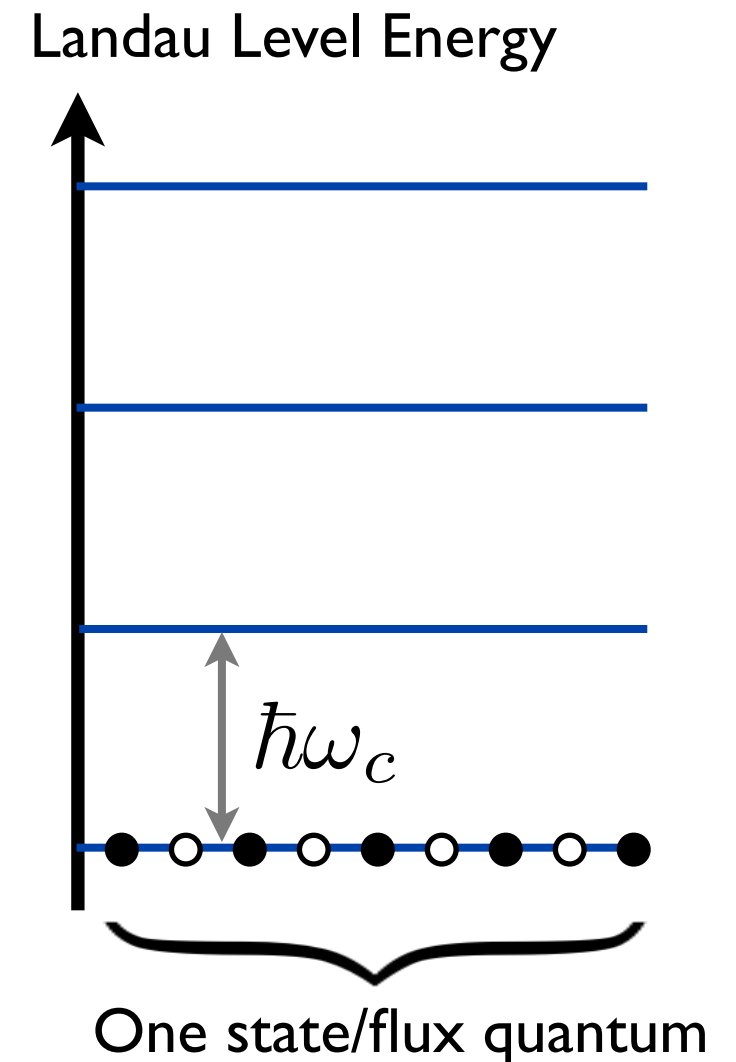
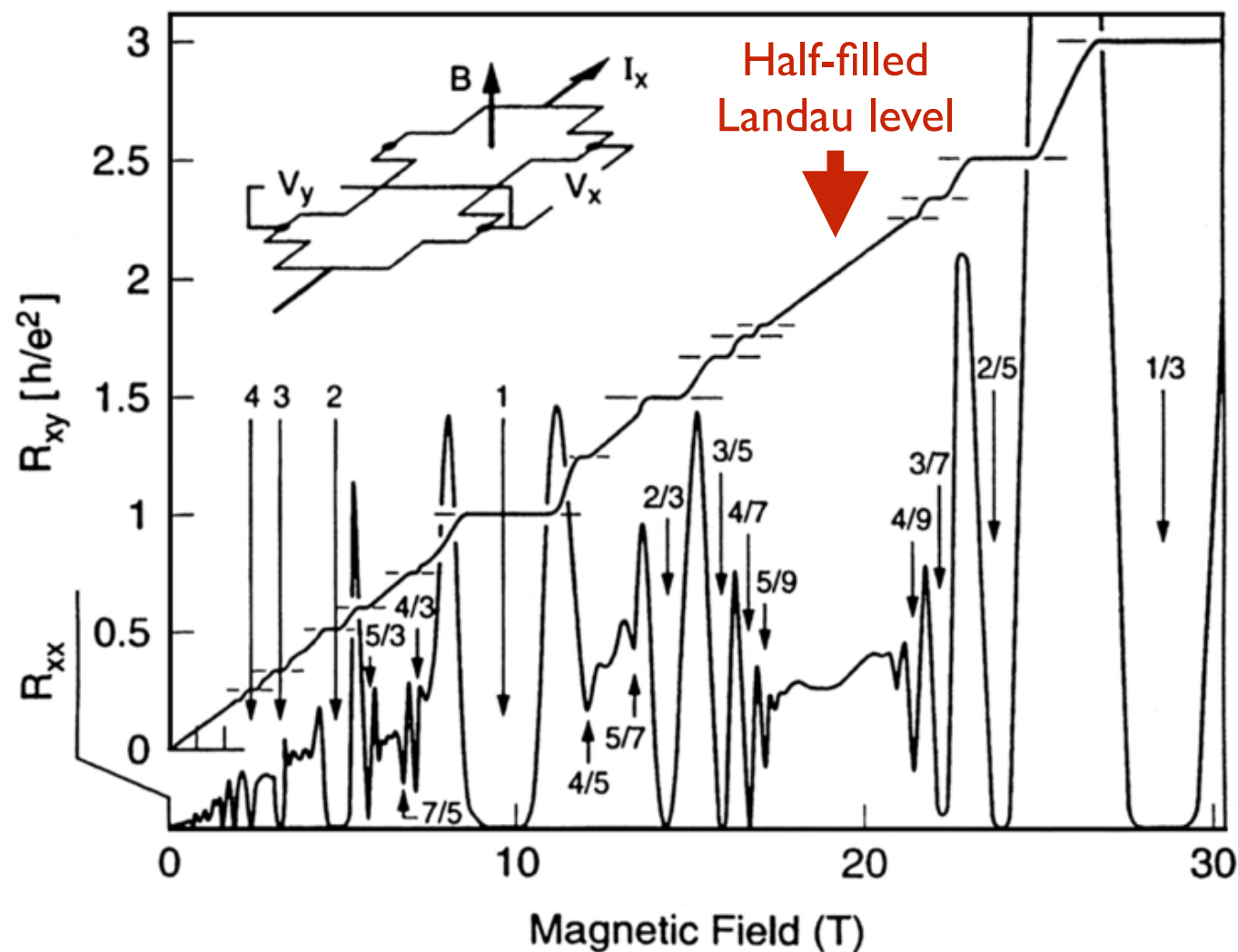


$O(100)$  plateaus observed to date. How might we most efficiently capture these topological phases?

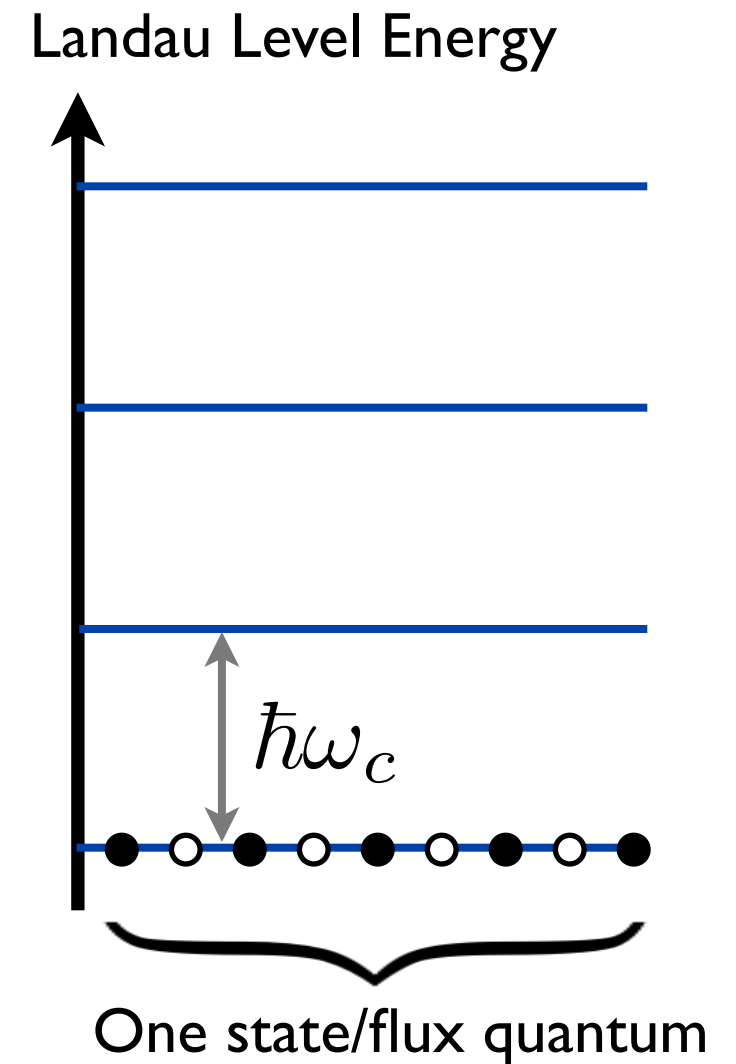
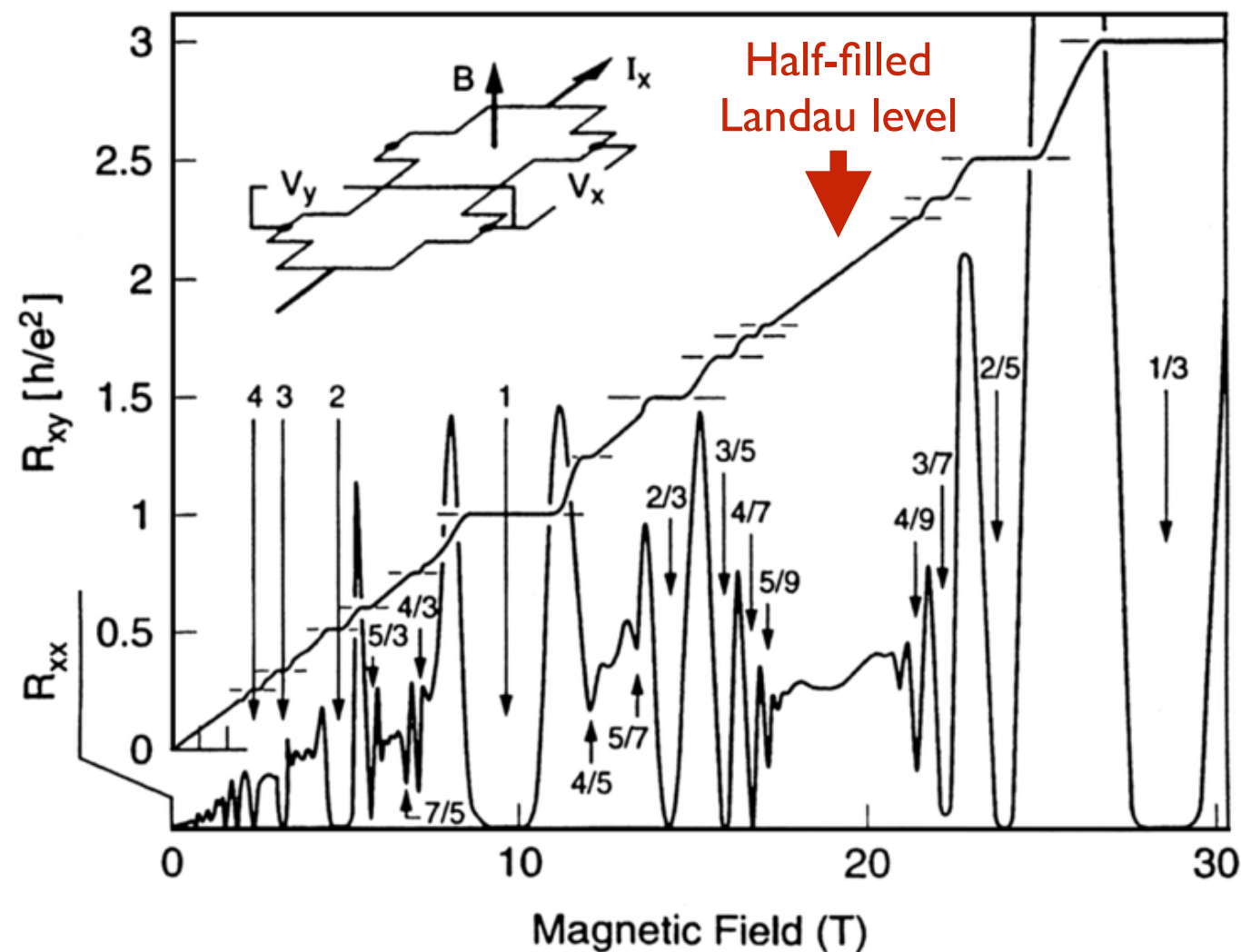


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# Composite Fermi liquid at $\nu = 1/2$

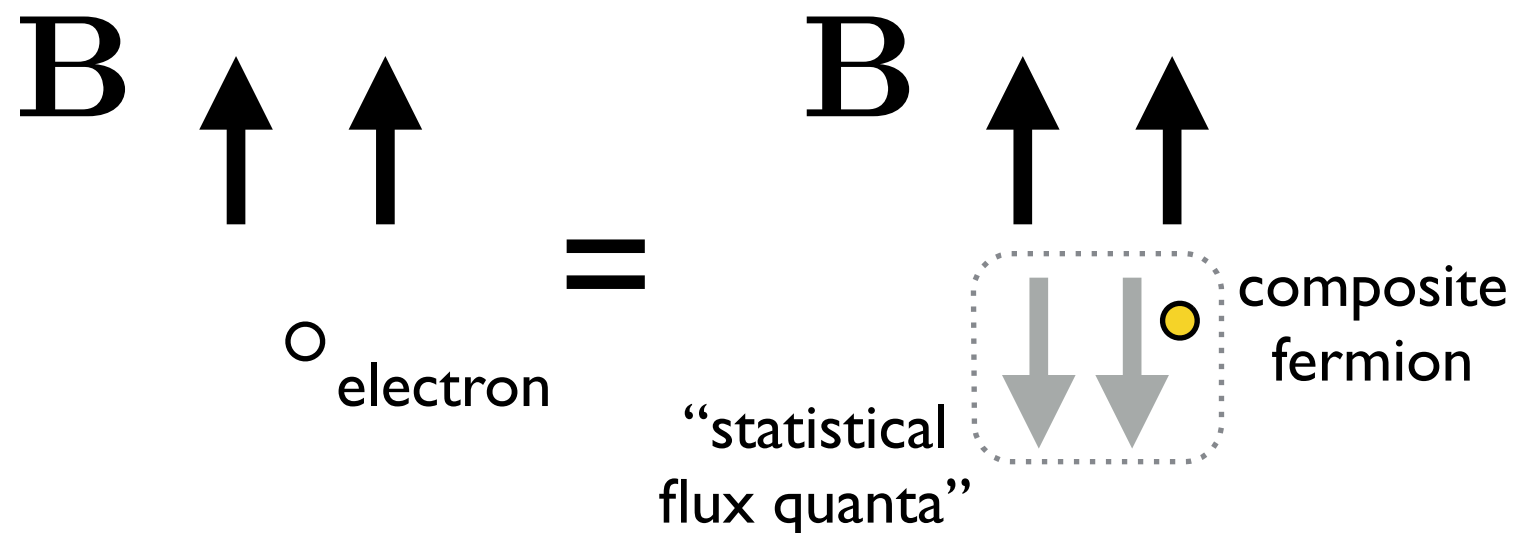
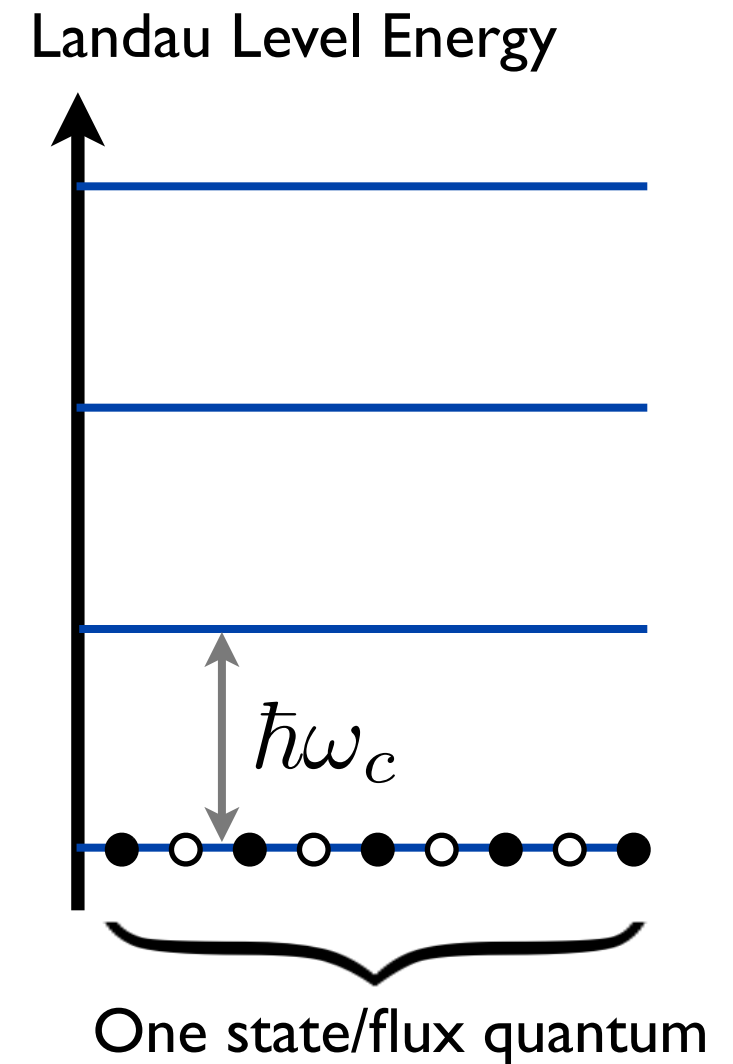
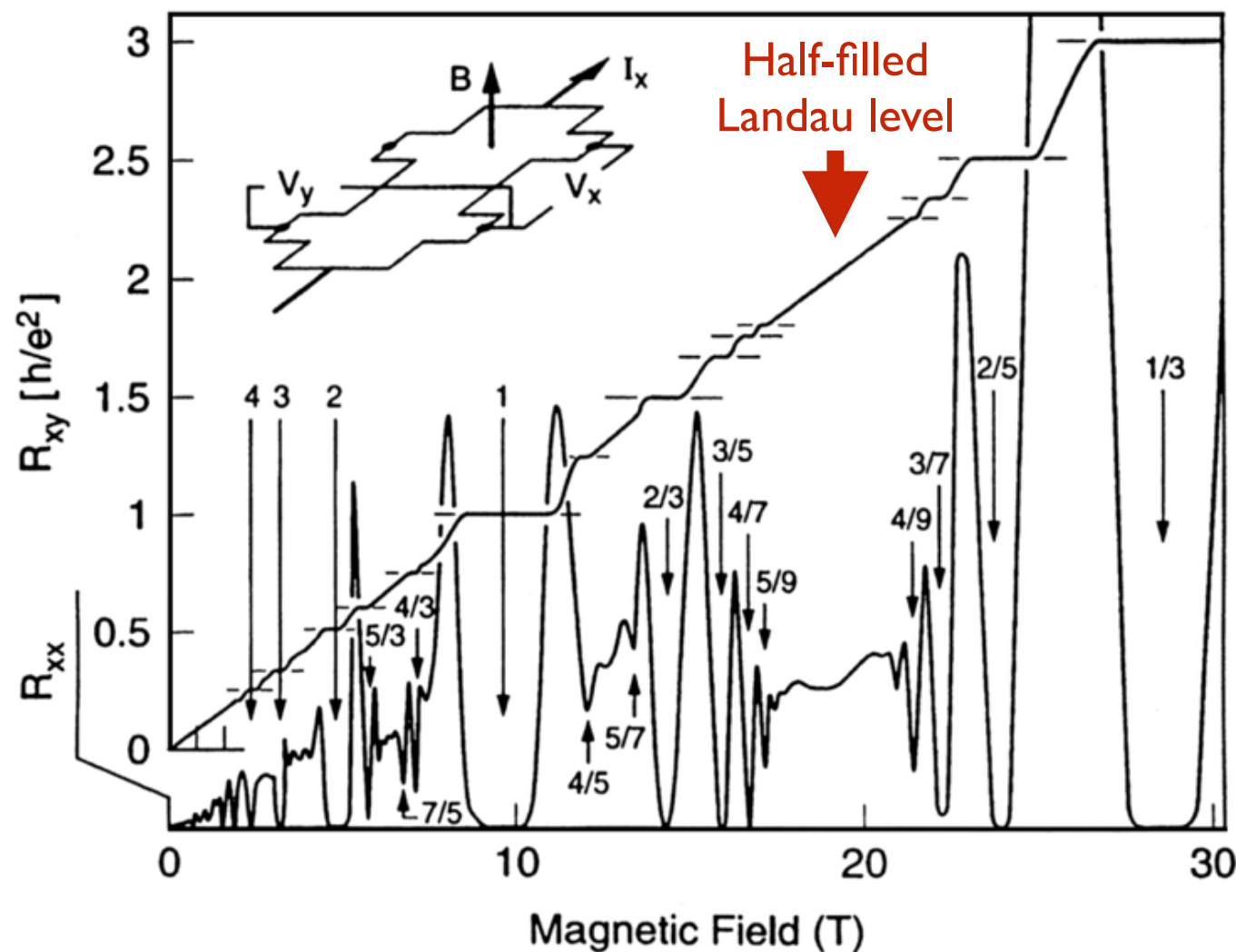


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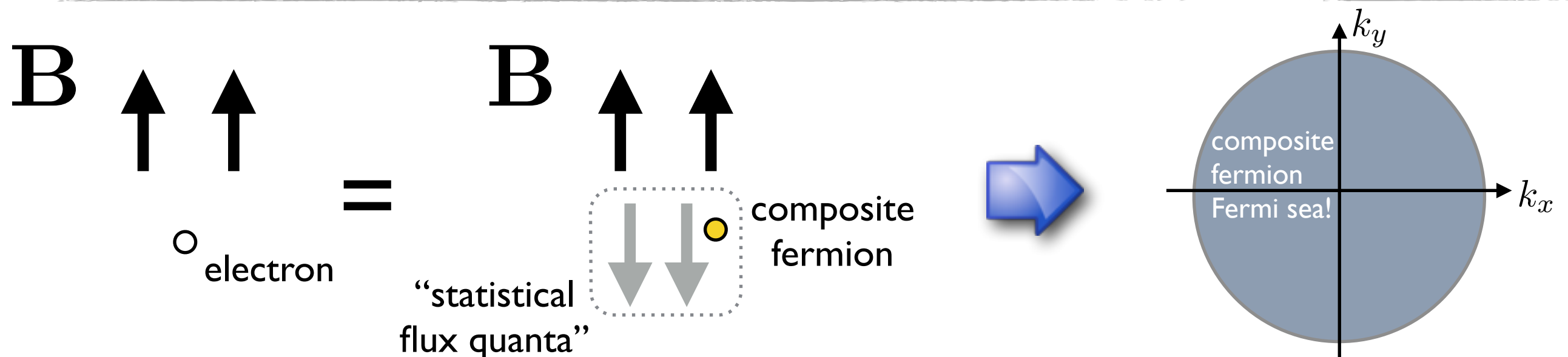
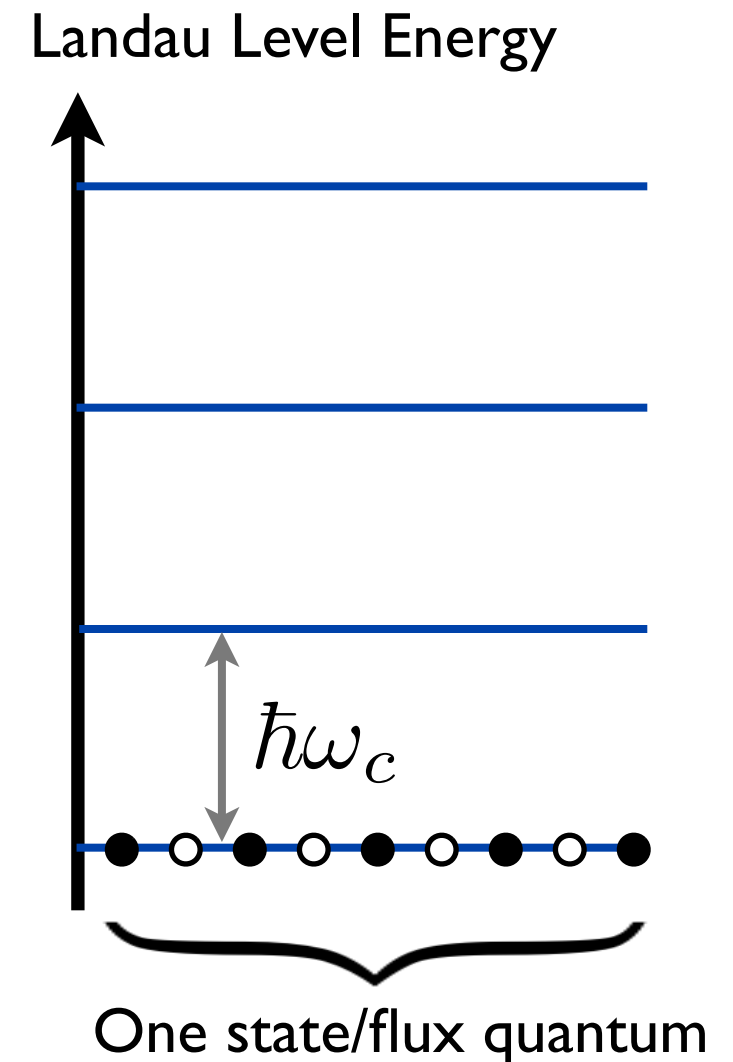
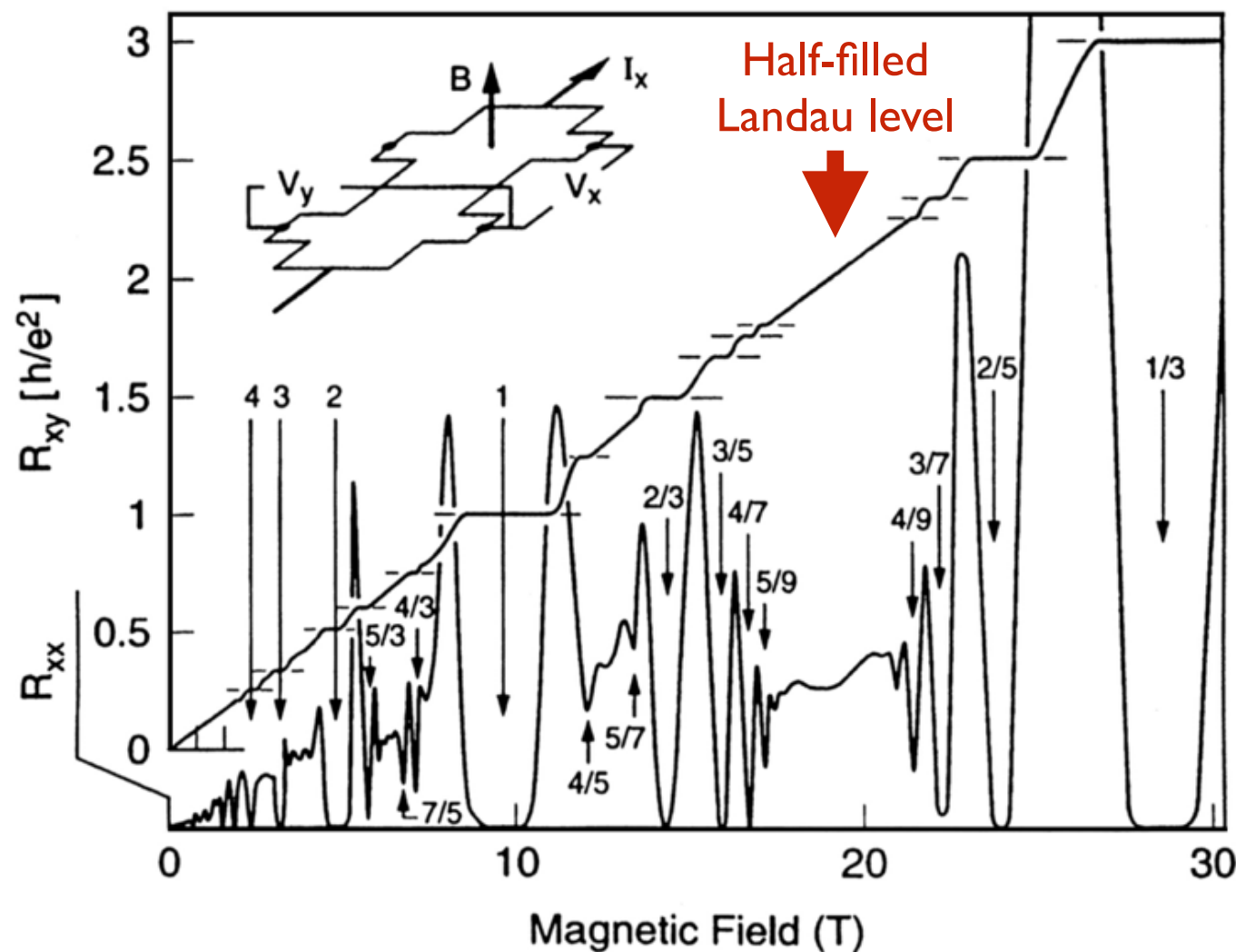
$\mathbf{B}$   $\uparrow$   $\uparrow$   
○ electron

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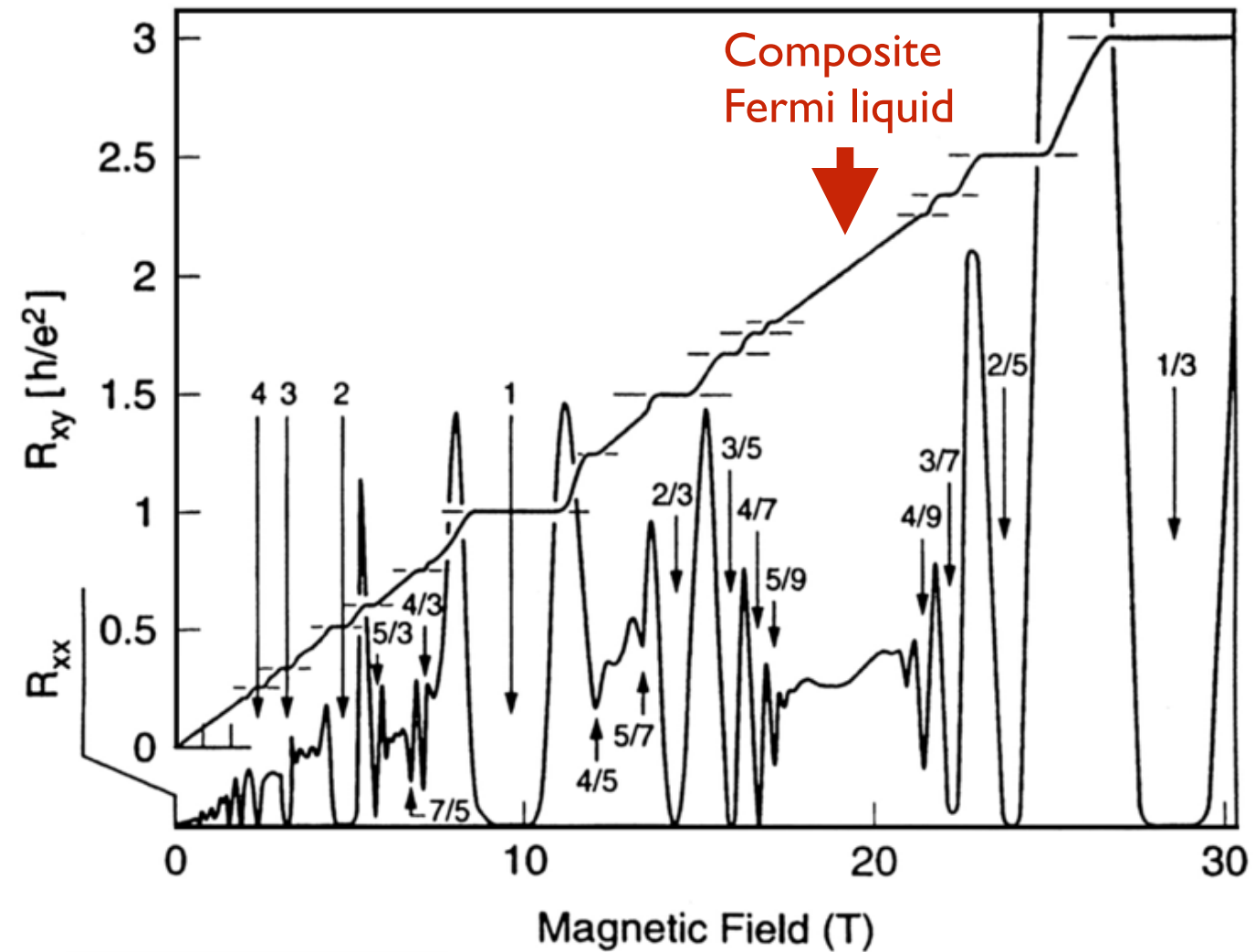
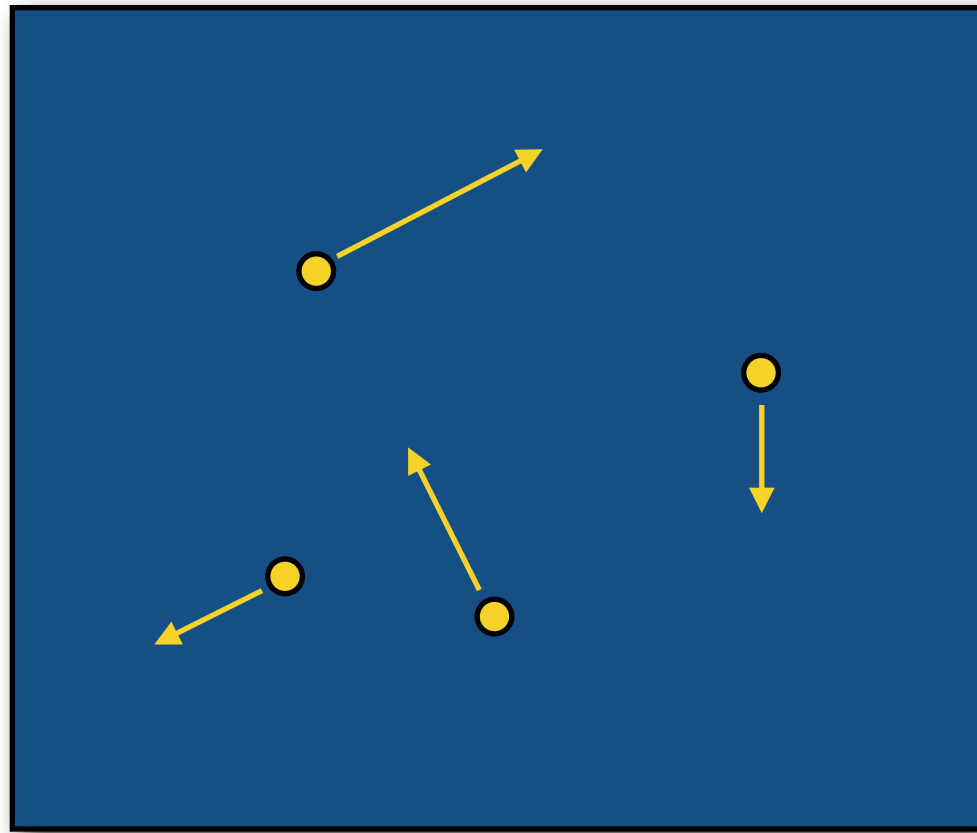


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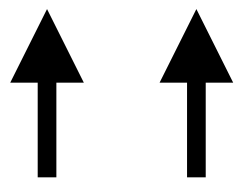


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$\odot$   
**B**

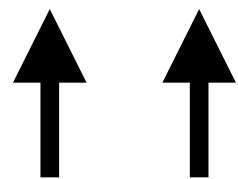


**B**



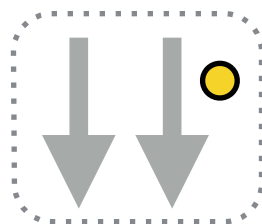
=

**B**

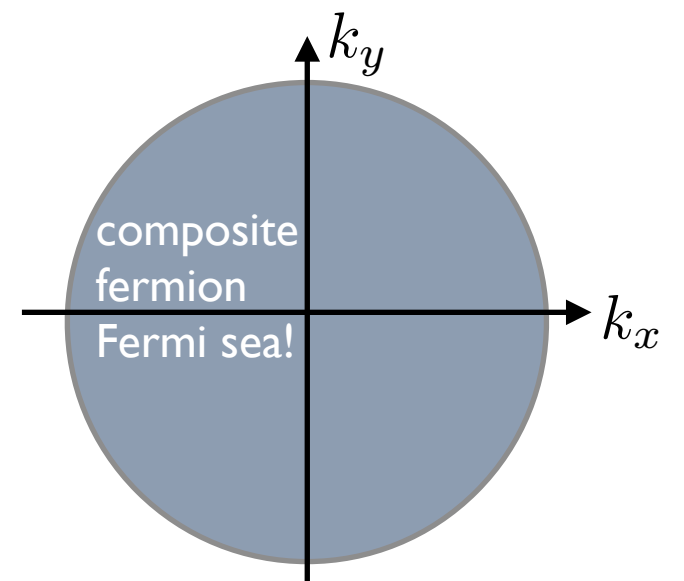


$\circ$   
electron

"statistical  
flux quanta"



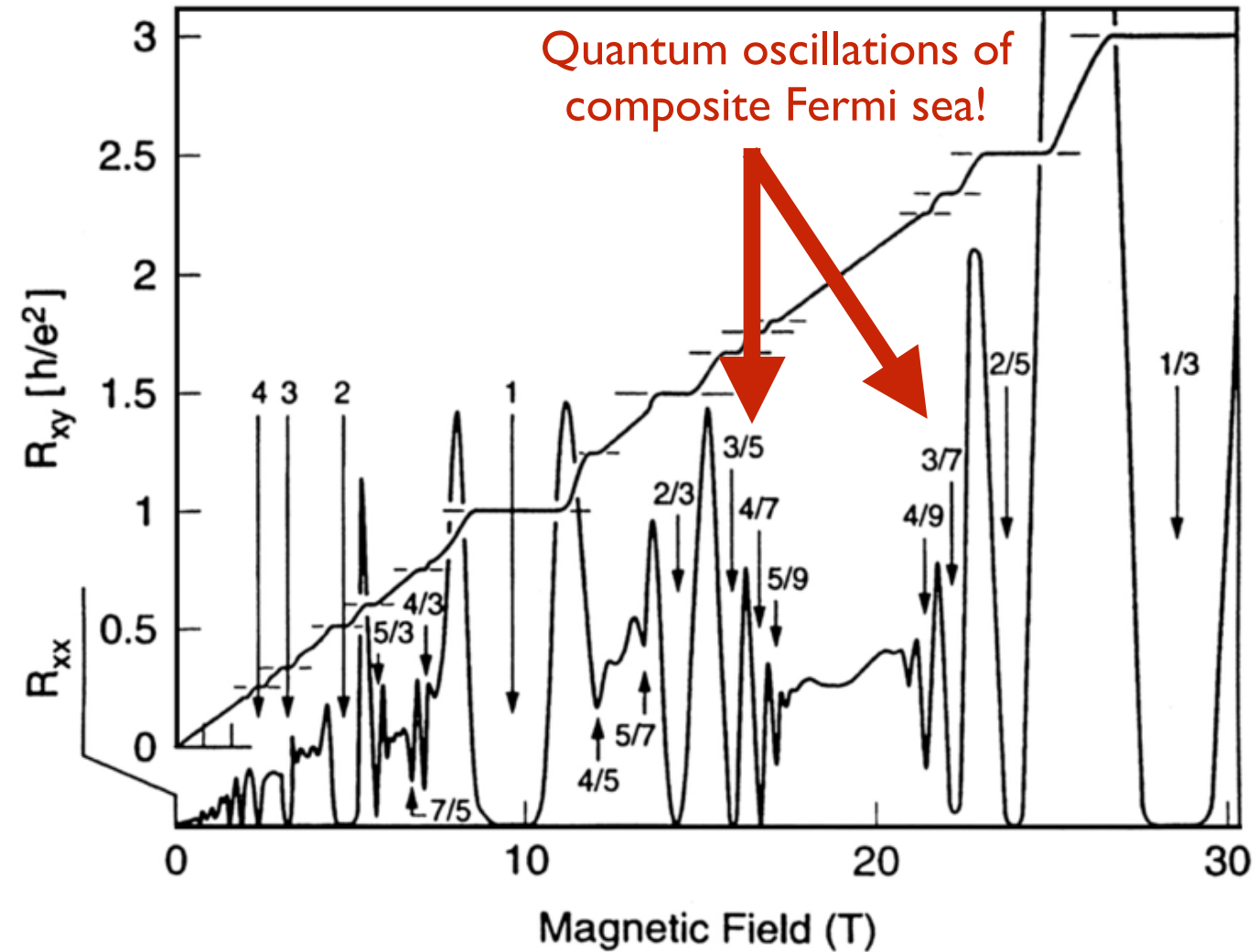
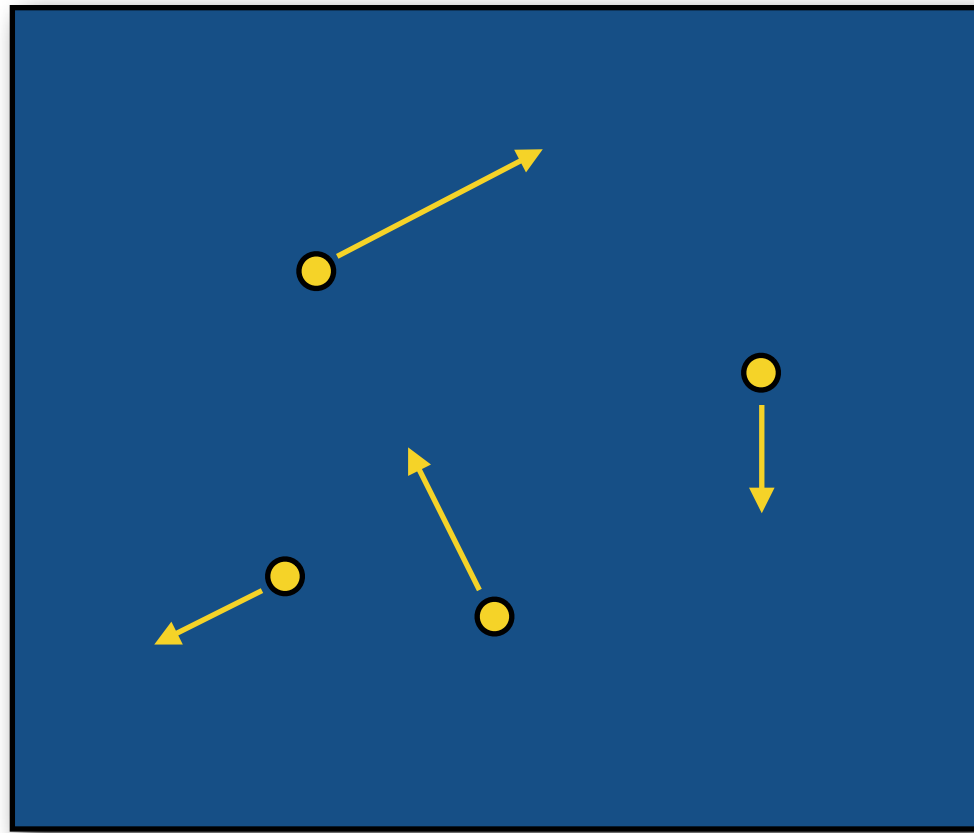
composite  
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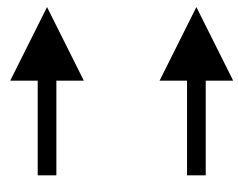


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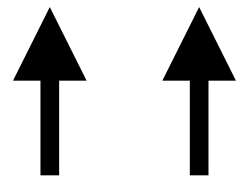


**B**



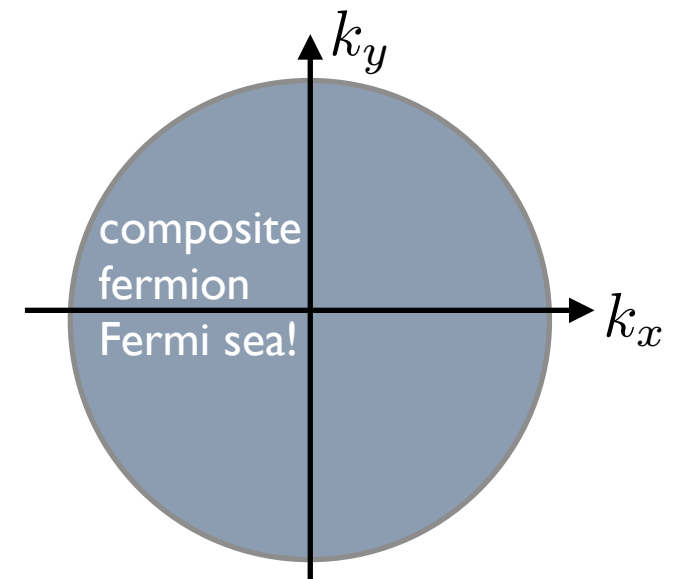
=

**B**



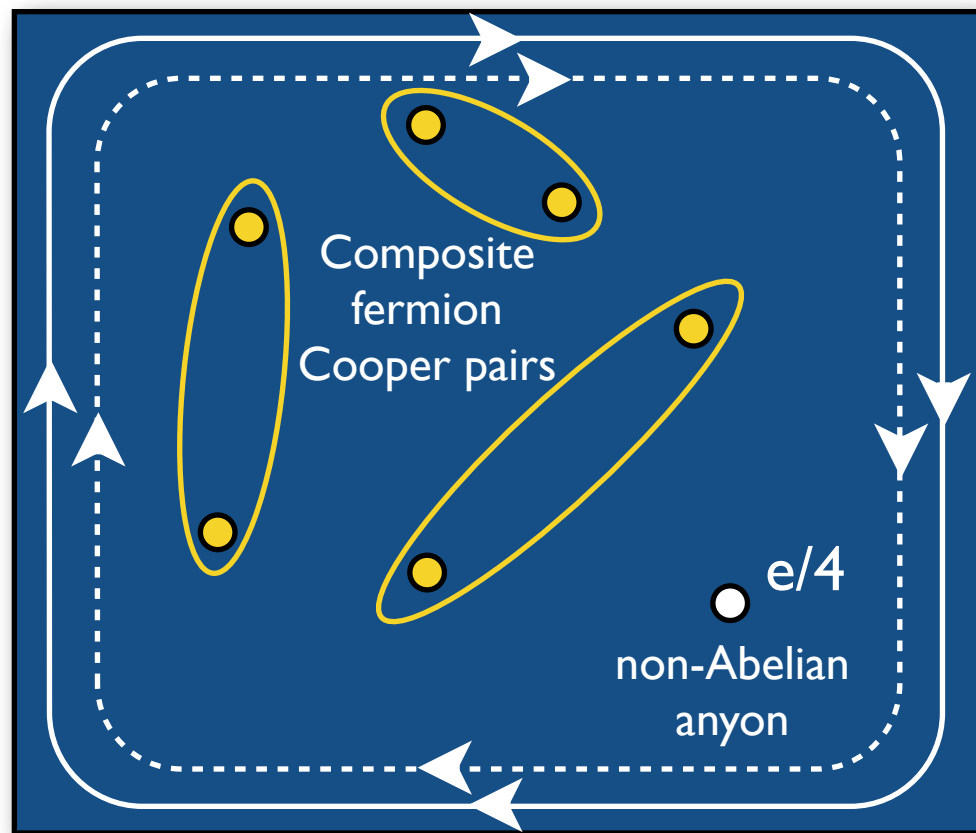
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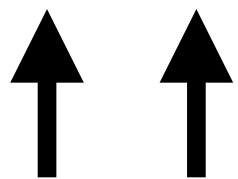
$\odot$   
**B**



co-propagating Majorana + charge edge modes

Pairing instability of composite Fermi sea yields **non-Abelian** “Moore-Read” fractional quantum Hall state!

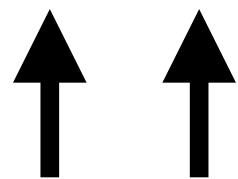
**B**



$\circ$  electron

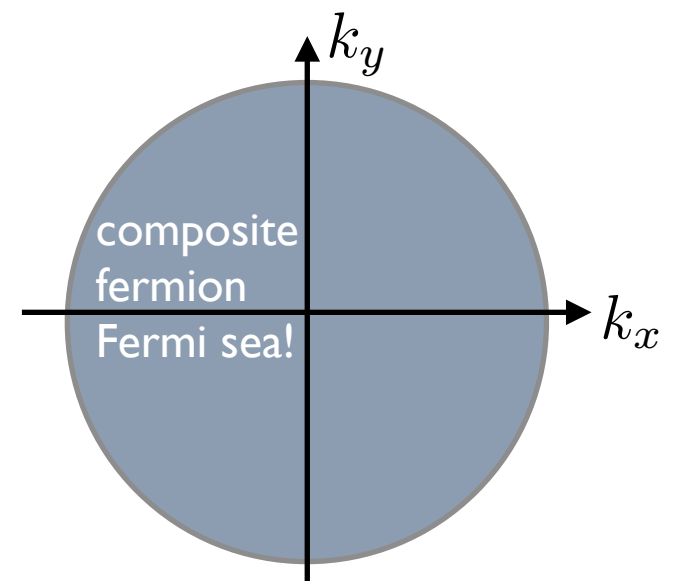
=

**B**



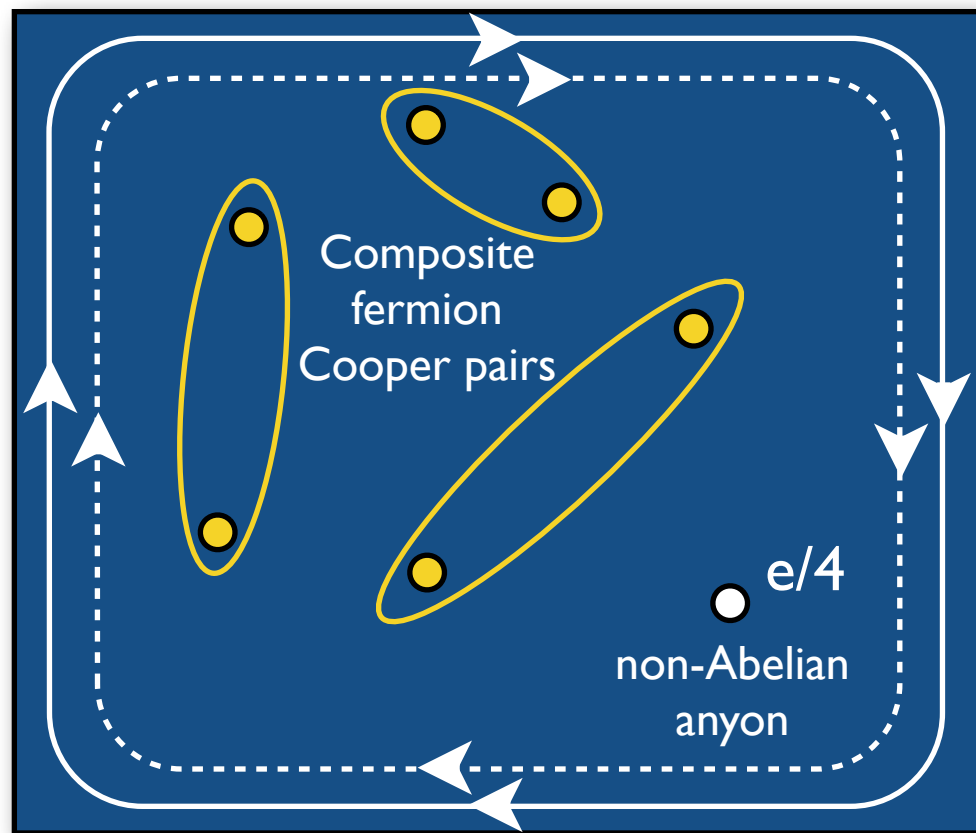
“statistical  
flux quanta”

composite  
fermion



# Composite Fermi liquid at $\nu = 1/2$

$\odot$   
**B**

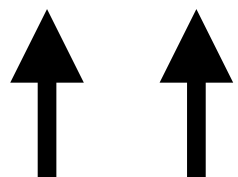


co-propagating Majorana + charge edge modes

Pairing instability of composite Fermi sea yields **non-Abelian** “Moore-Read” fractional quantum Hall state!

Moral: emergent “metals” are interesting and provide efficient way of capturing exotic topologically ordered gapped phases.

**B**

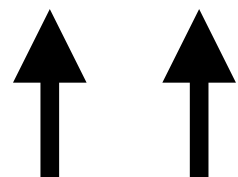


$\circ$

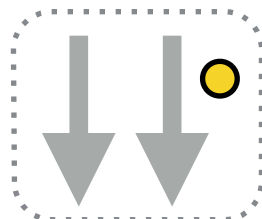
electron

=

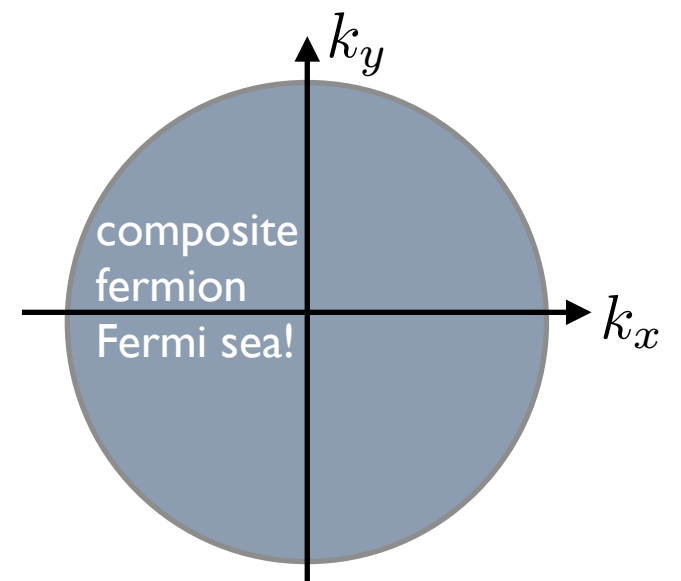
**B**



“statistical  
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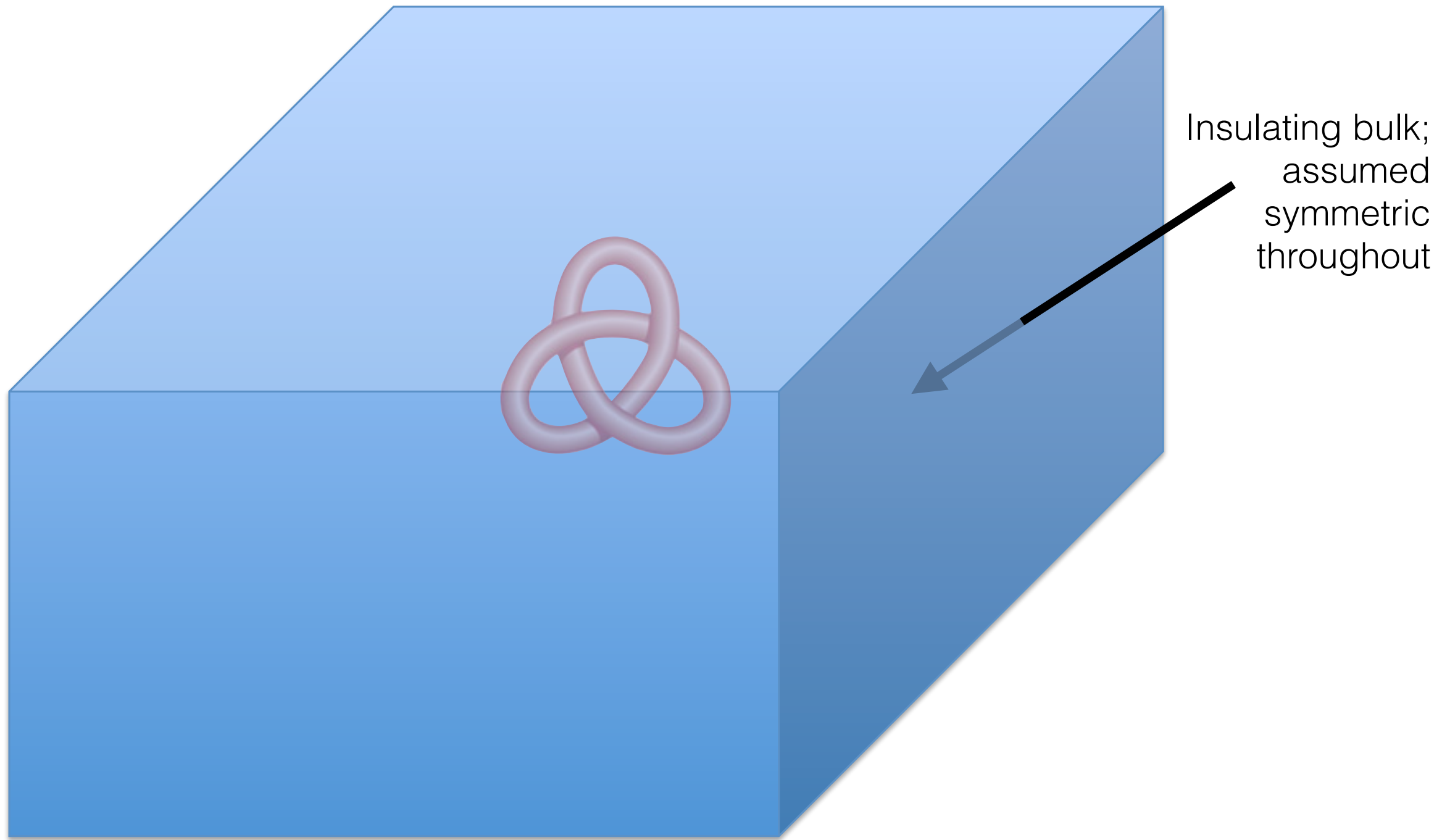


composite  
fermion



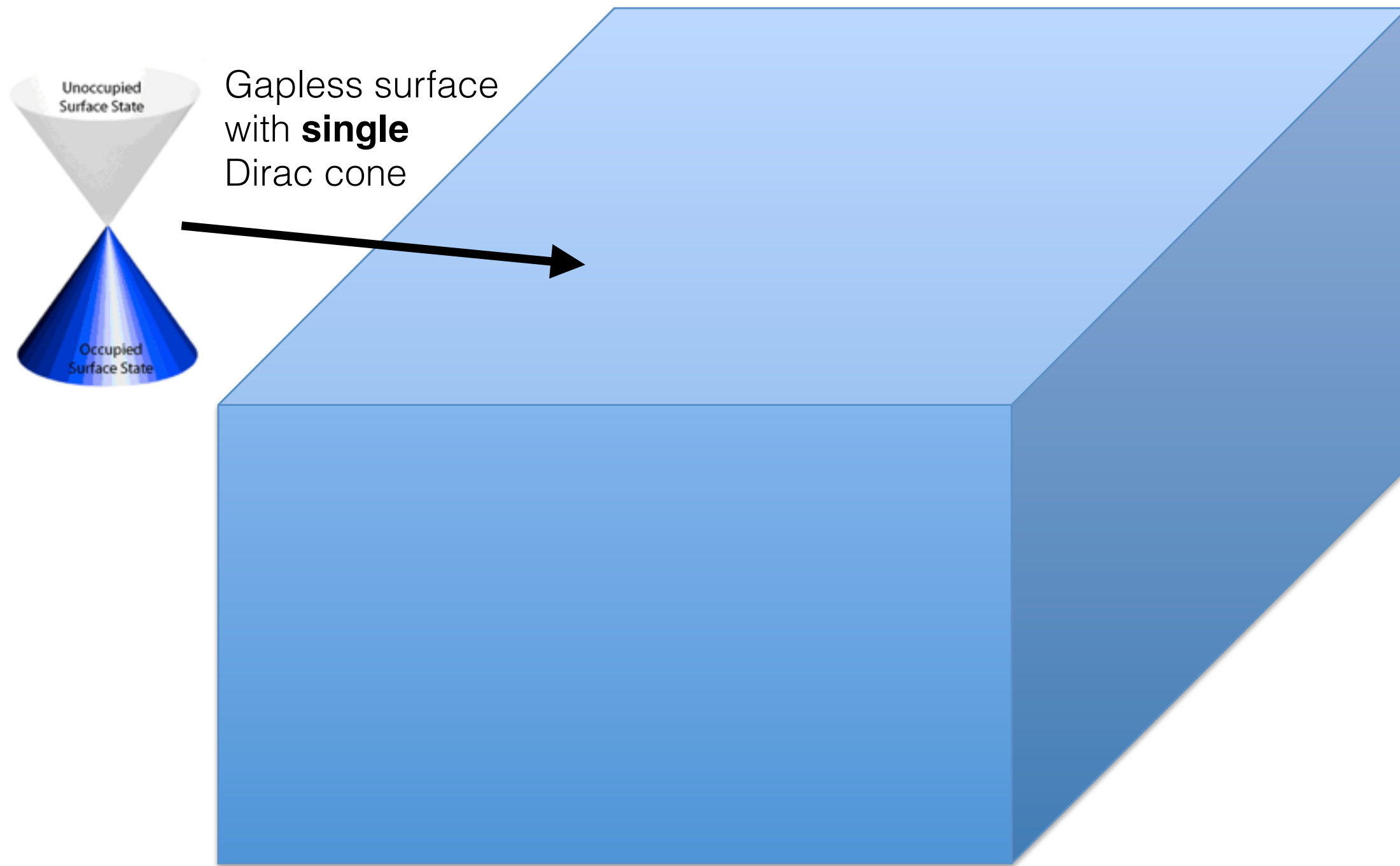
# Anomalous 3D TI surface physics

3D topological insulator: protected by **time-reversal, U(1) particle conservation symmetries**



# Anomalous 3D TI surface physics

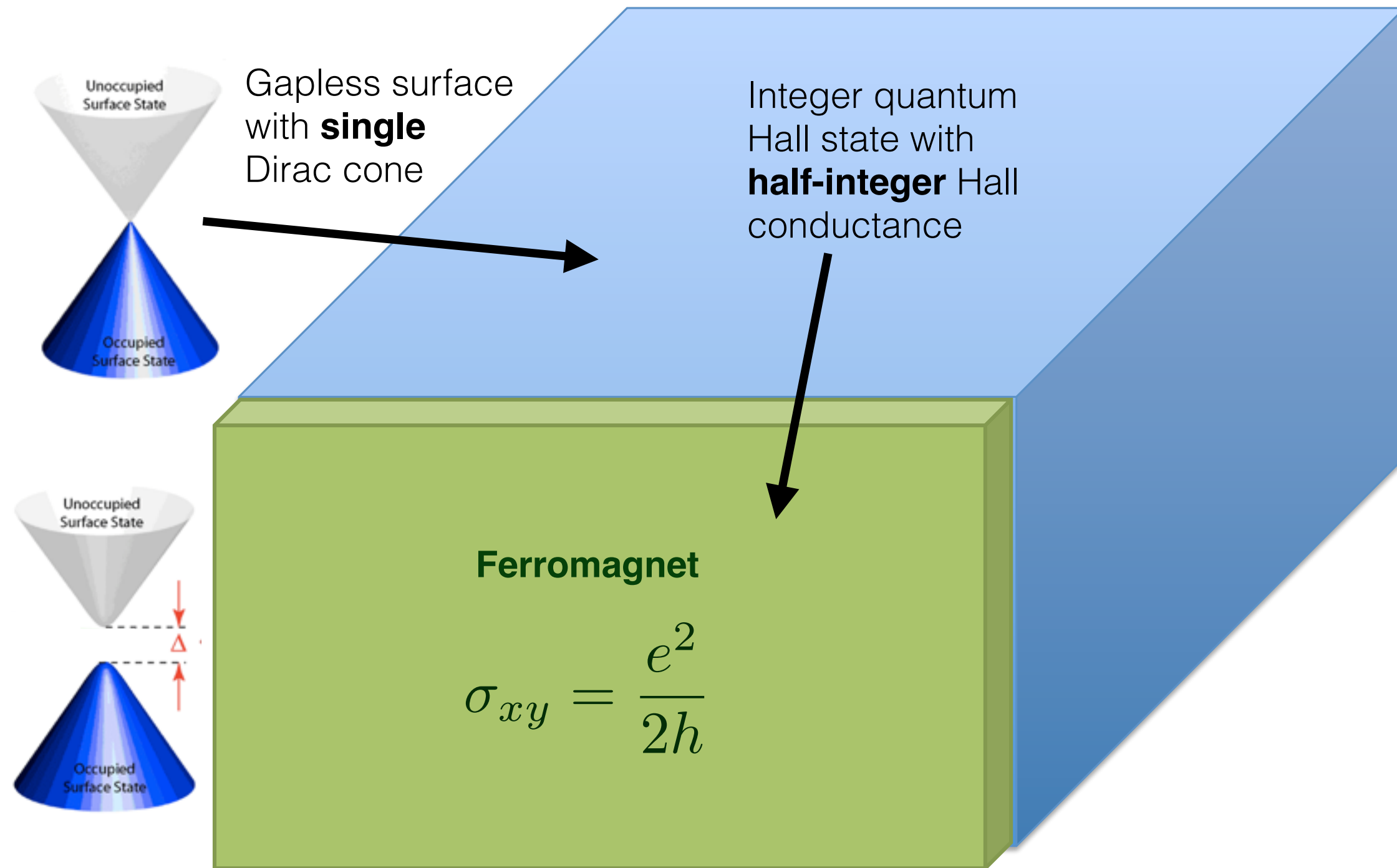
3D topological insulator: protected by **time-reversal, U(1) particle conservation symmetries**



Impossible band structure in strict 2D systems with time-reversal, where only **even** # of Dirac cones can appear.

# Anomalous 3D TI surface physics

3D topological insulator: protected by **time-reversal, U(1) particle conservation symmetries**

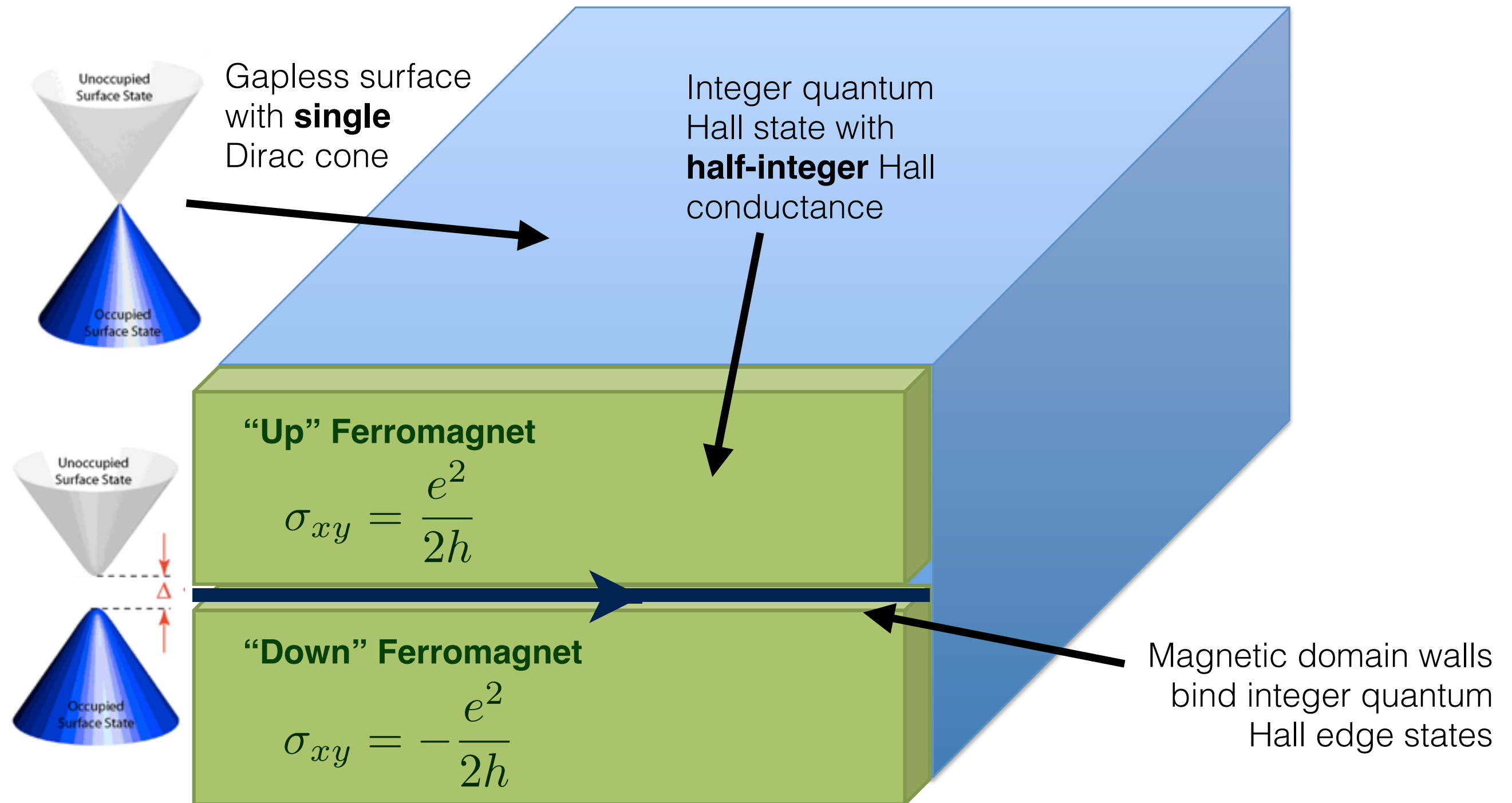


Impossible in strict 2D, since would imply fractionalization, topological order.



# Anomalous 3D TI surface physics

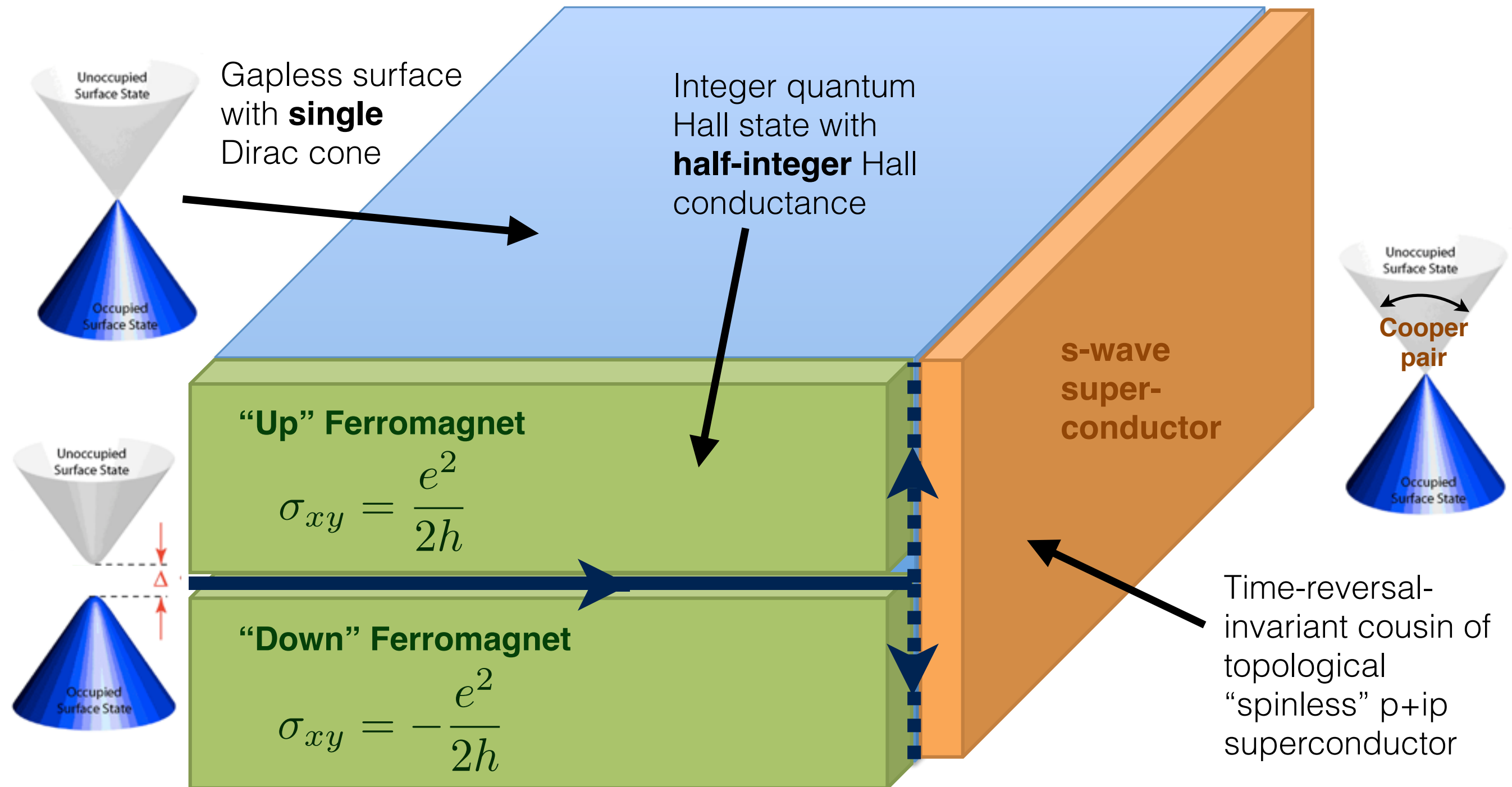
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# Anomalous 3D TI surface physics

3D topological insulator: protected by **time-reversal, U(1) particle conservation symmetries**



Impossible in strict 2D, since superconductor would always break T.

Without interactions, this is the full story:

- Symmetry implies massless electron Dirac cone
- broken symmetry implies anomalous gapped phases.

Does symmetry imply surface metallicity more generally?

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## Does symmetry imply surface metallicity more generally? No!

### **A Time-Reversal Invariant Topological Phase at the Surface of a 3D Topological Insulator**

Parsa Bonderson,<sup>1</sup> Chetan Nayak,<sup>1,2</sup> and Xiao-Liang Qi<sup>3,1</sup>

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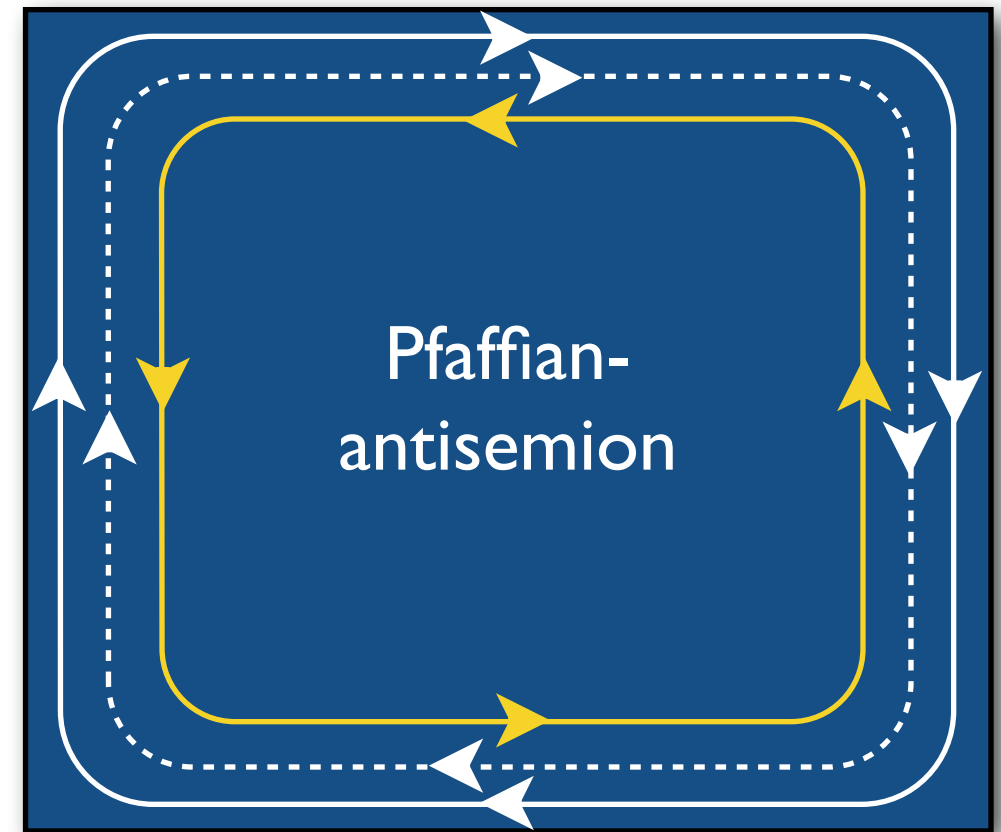
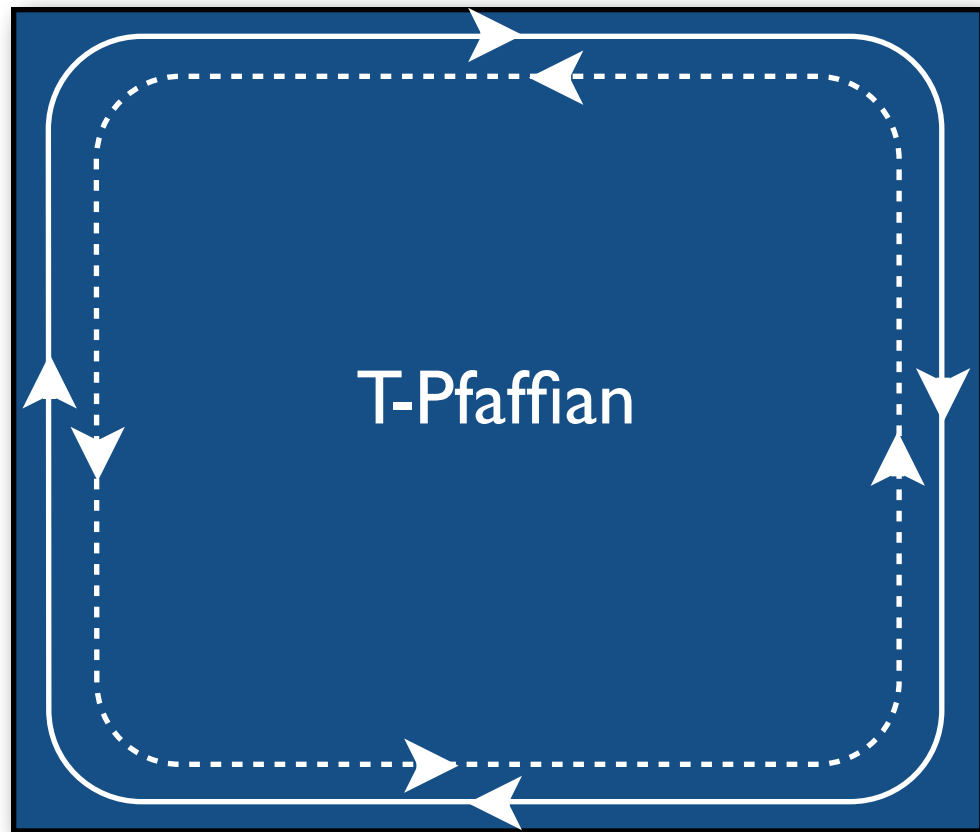
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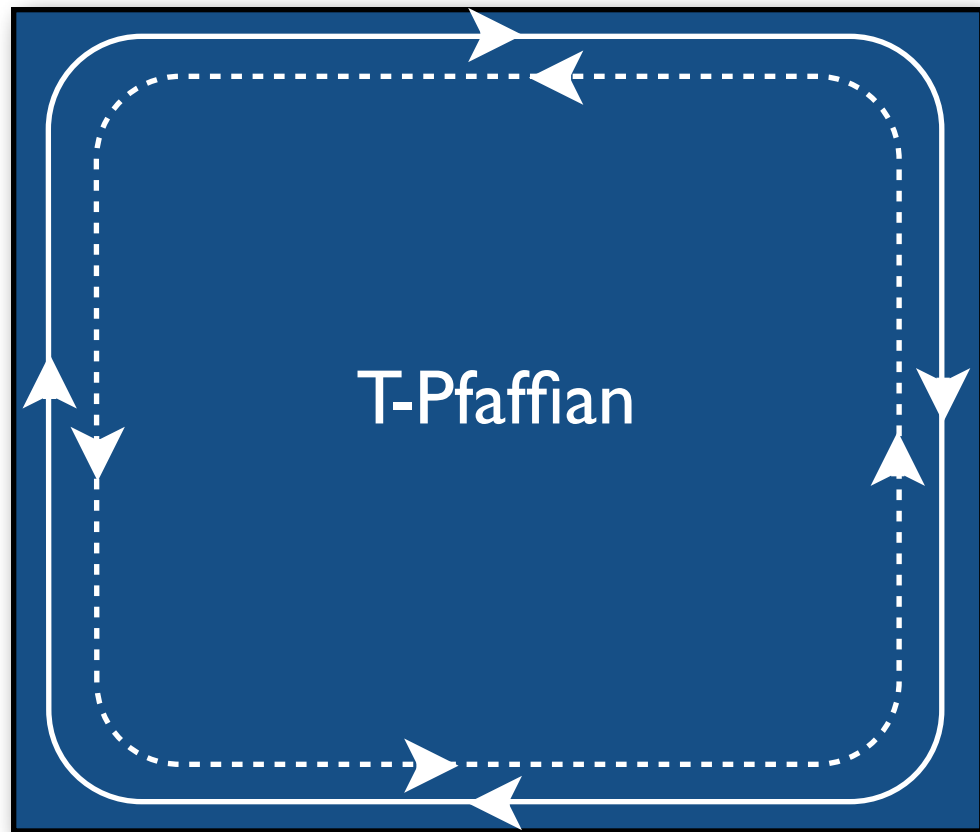
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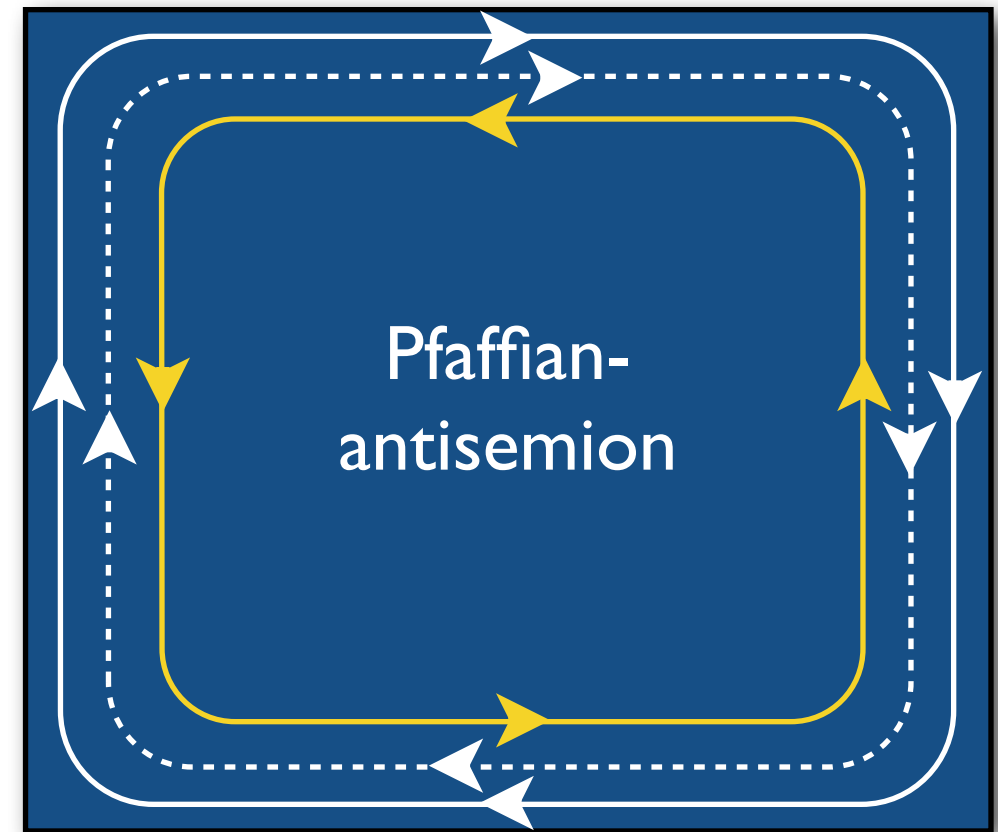
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Clearly break  
time reversal  
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2D, but **not** on  
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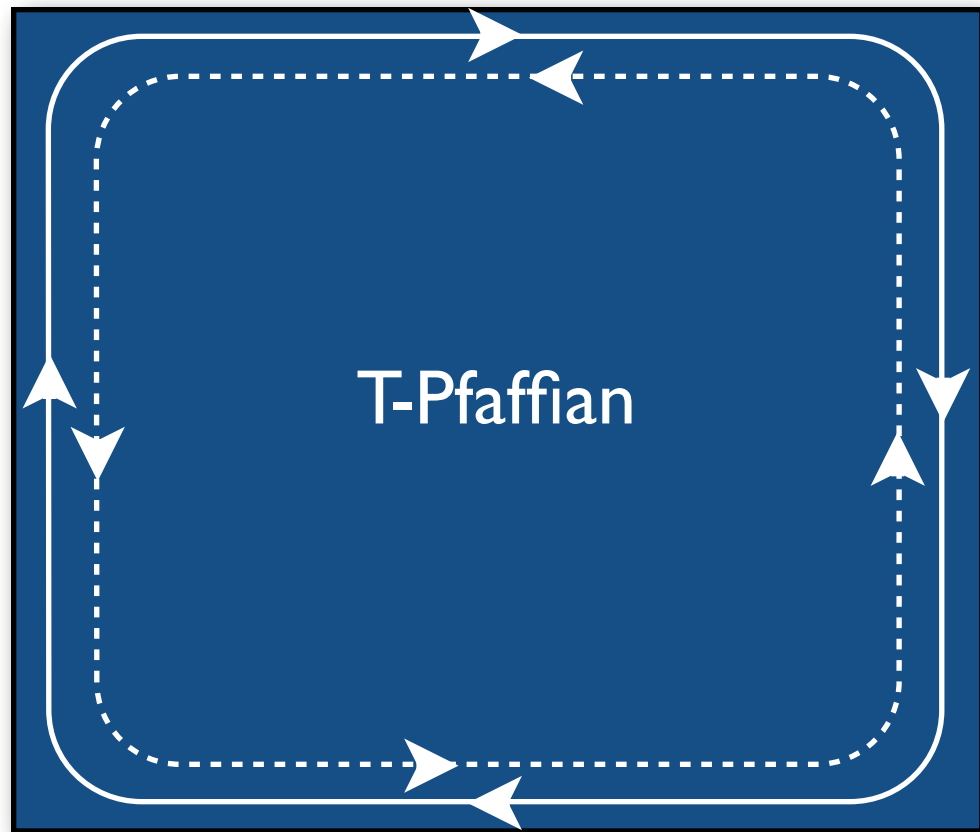
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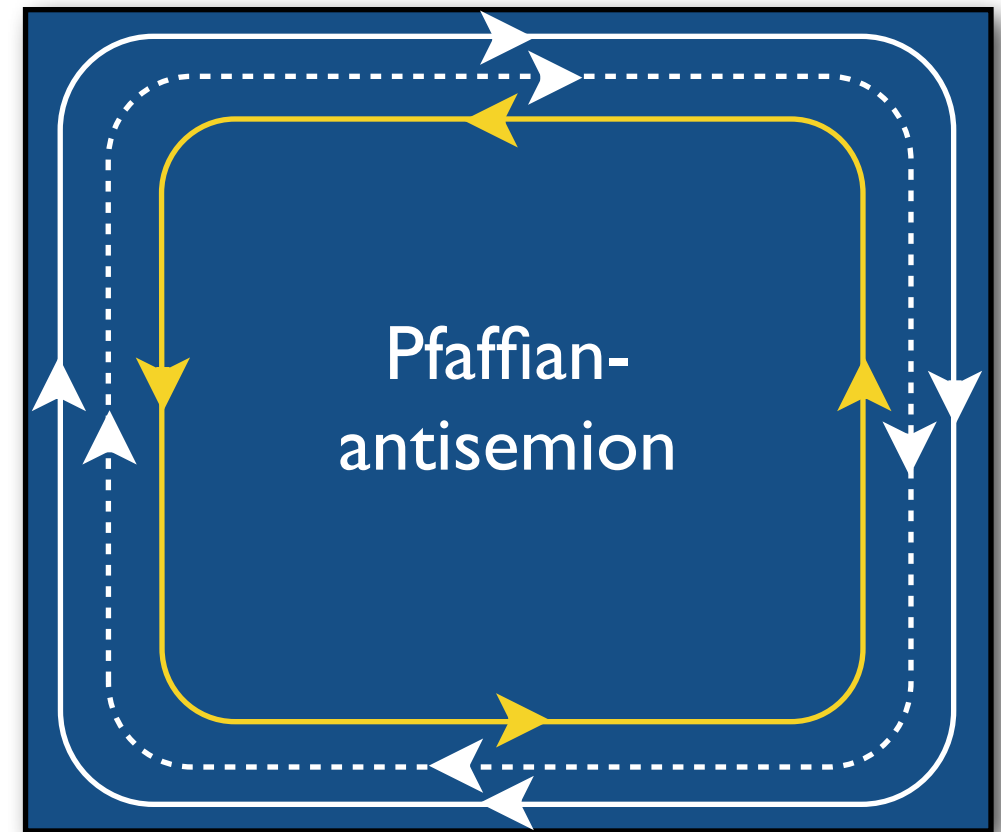
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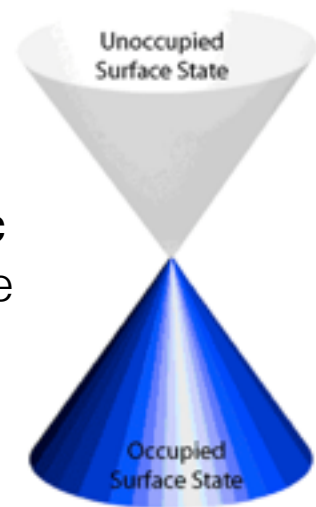
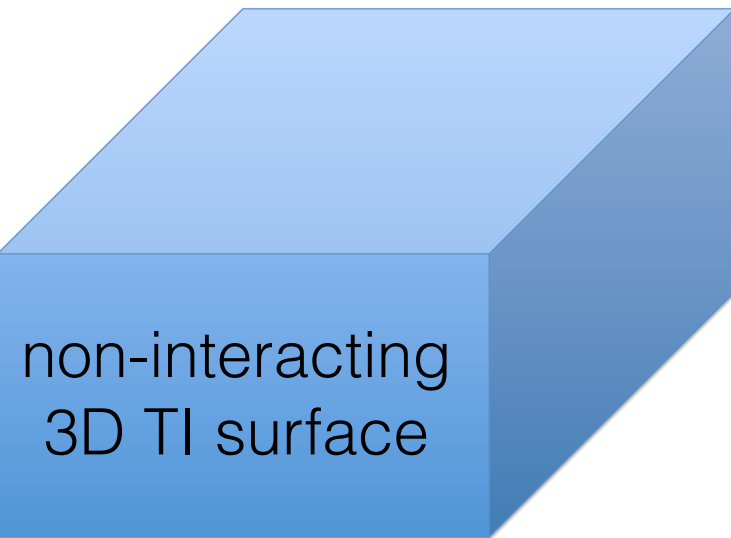
Clearly break  
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**“Composite Dirac  
liquids”** = new correlated  
gapless surface states  
that unify these works.

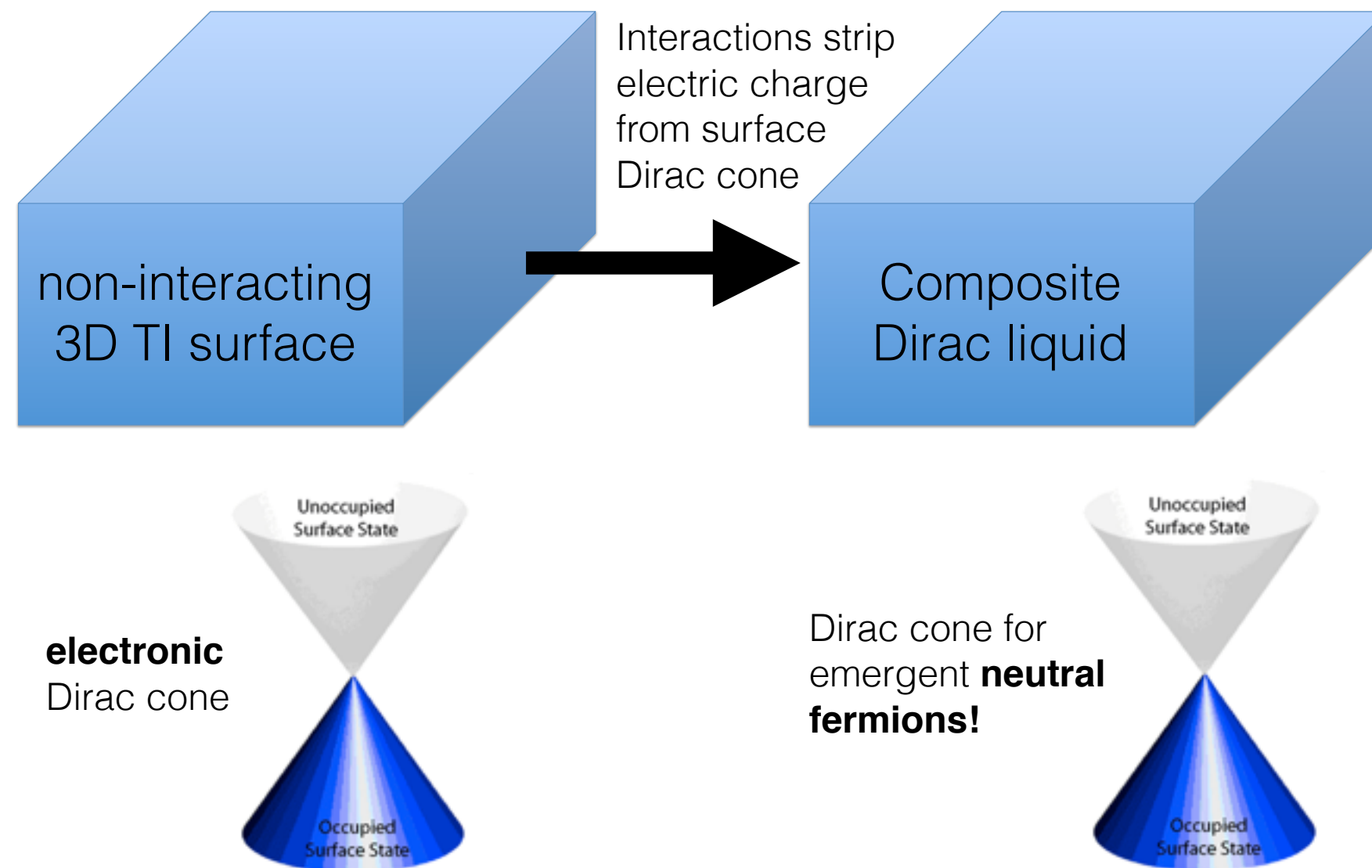
# Composite Dirac liquids: phenomenology

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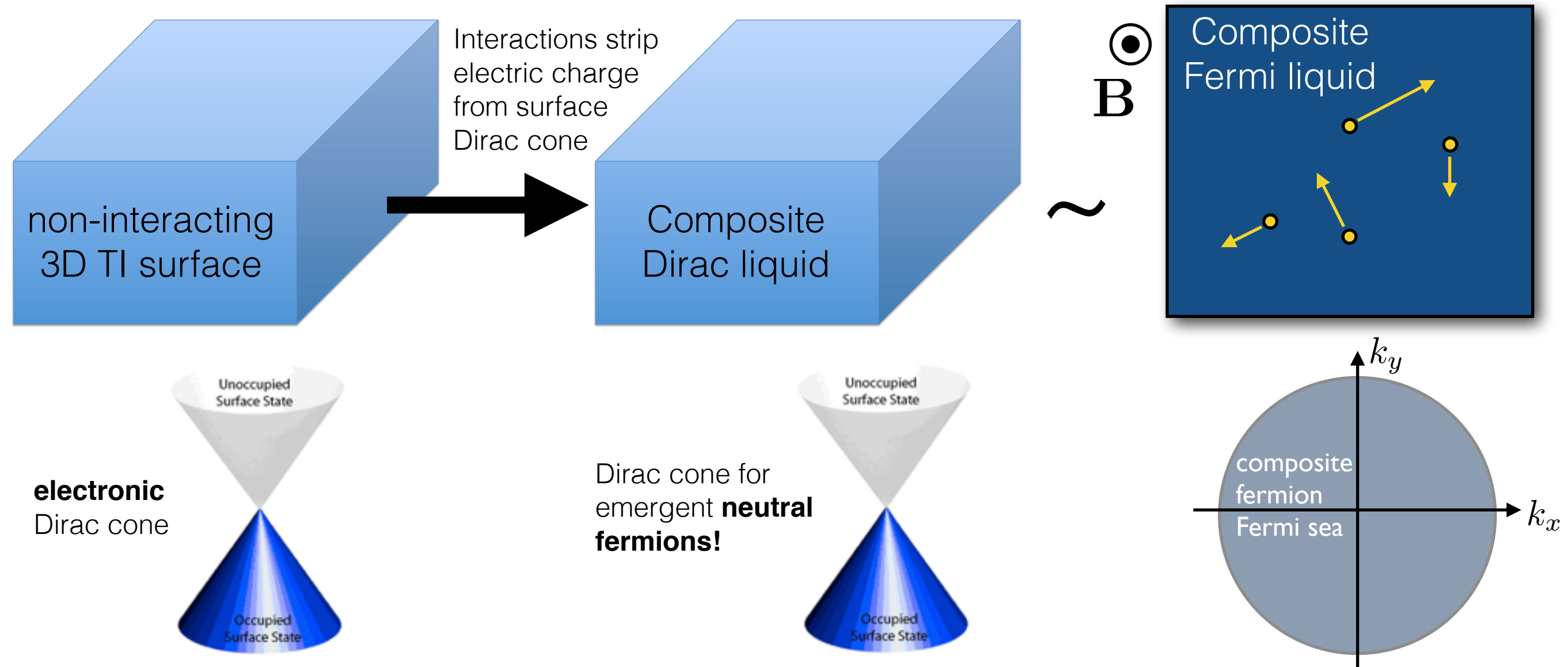
**electronic**  
Dirac cone

# Composite Dirac liquids: phenomenology



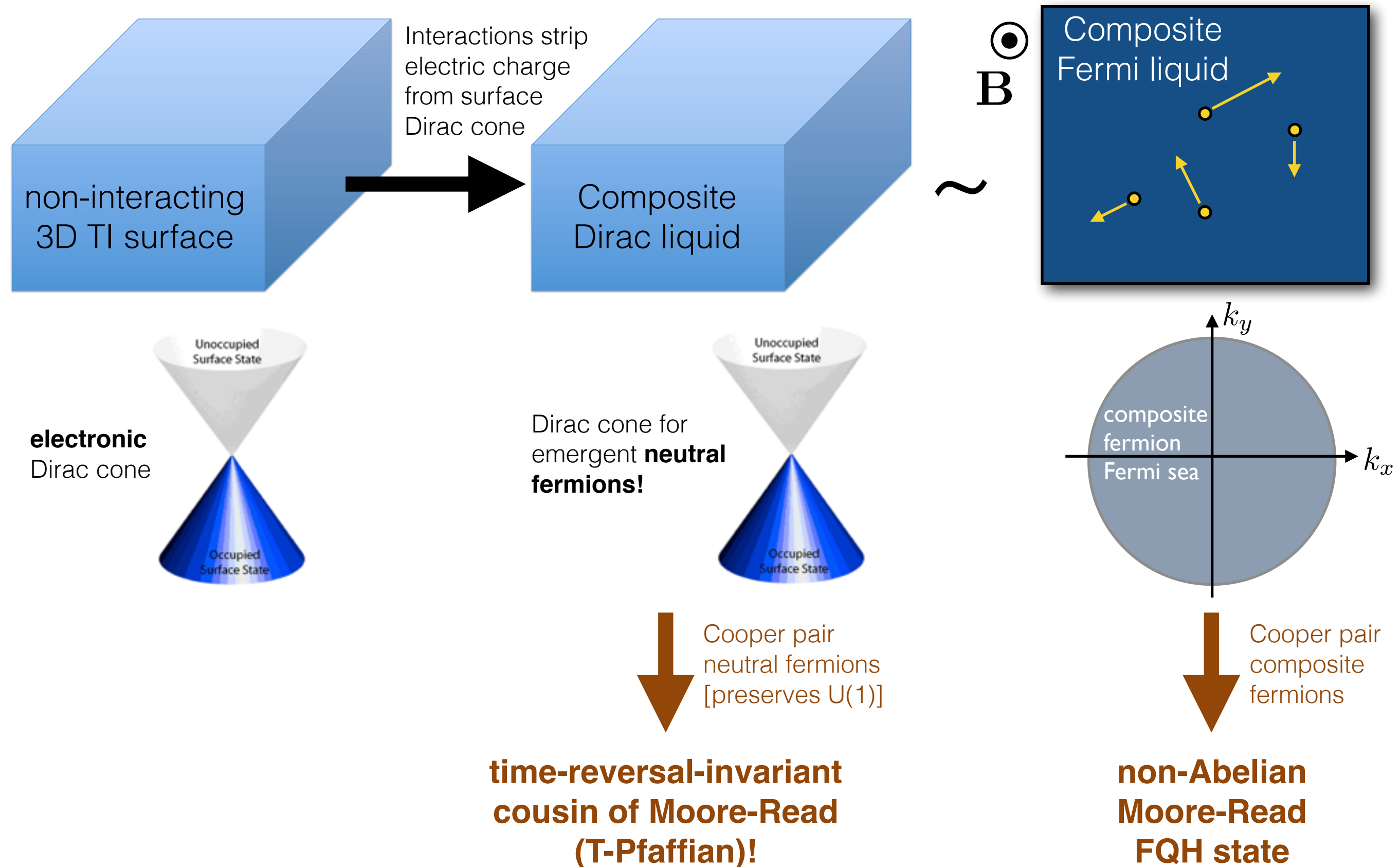
- Charge sector gapped, so surface is electrical insulator
- Neutral fermions yield “metallic” thermal transport similar to original Dirac cone
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# Composite Dirac liquids: phenomenology



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# Composite Dirac liquids: phenomenology



# Accessing composite Dirac liquids

---

In an ideal world:  $H_{\text{Dirac}} + H_{\text{interactions}} \Rightarrow$  Composite Dirac liquid



3D TI surface



# Accessing composite Dirac liquids

---

In an ideal world:  ~~$H_{\text{Dirac}} + H_{\text{interactions}} \Rightarrow$~~  Composite Dirac liquid (too hard)



3D TI surface

Instead, will “cheat” and follow route with virtues of physical transparency, analytical control.

# Accessing composite Dirac liquids

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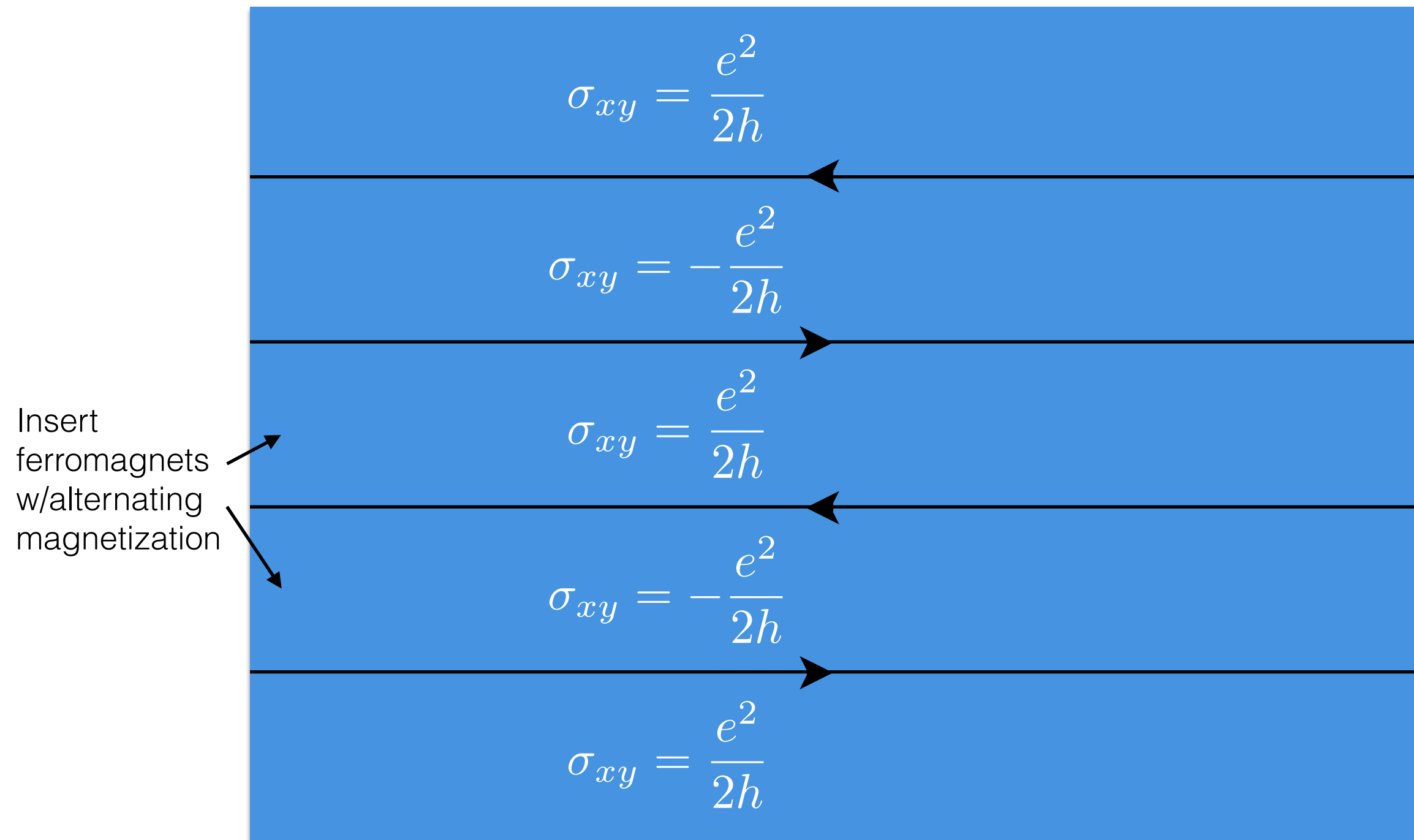
Key technical idea: relax time-reversal to weaker “antiferromagnetic” symmetry  
(states *still* impossible in 2D)



3D TI surface

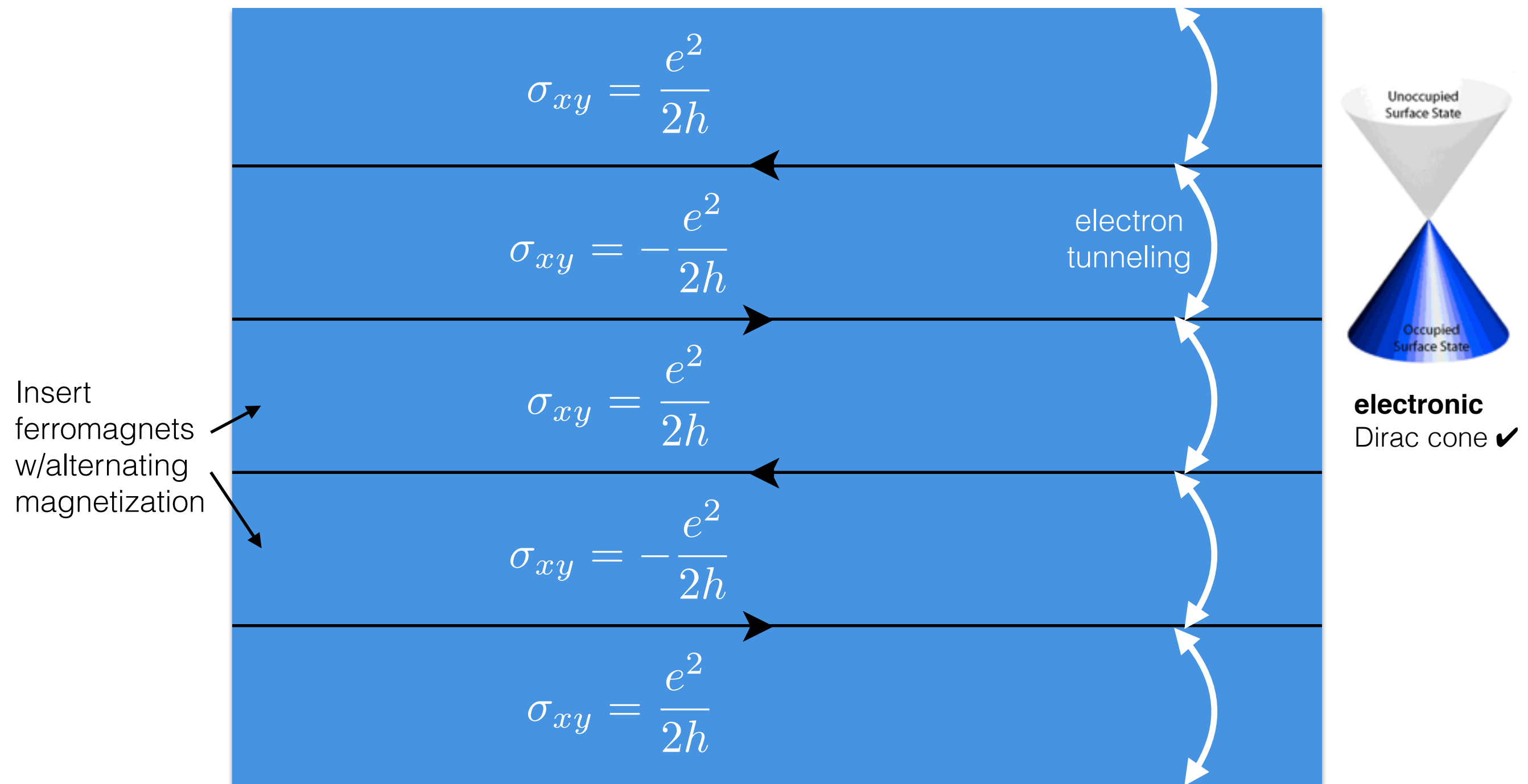
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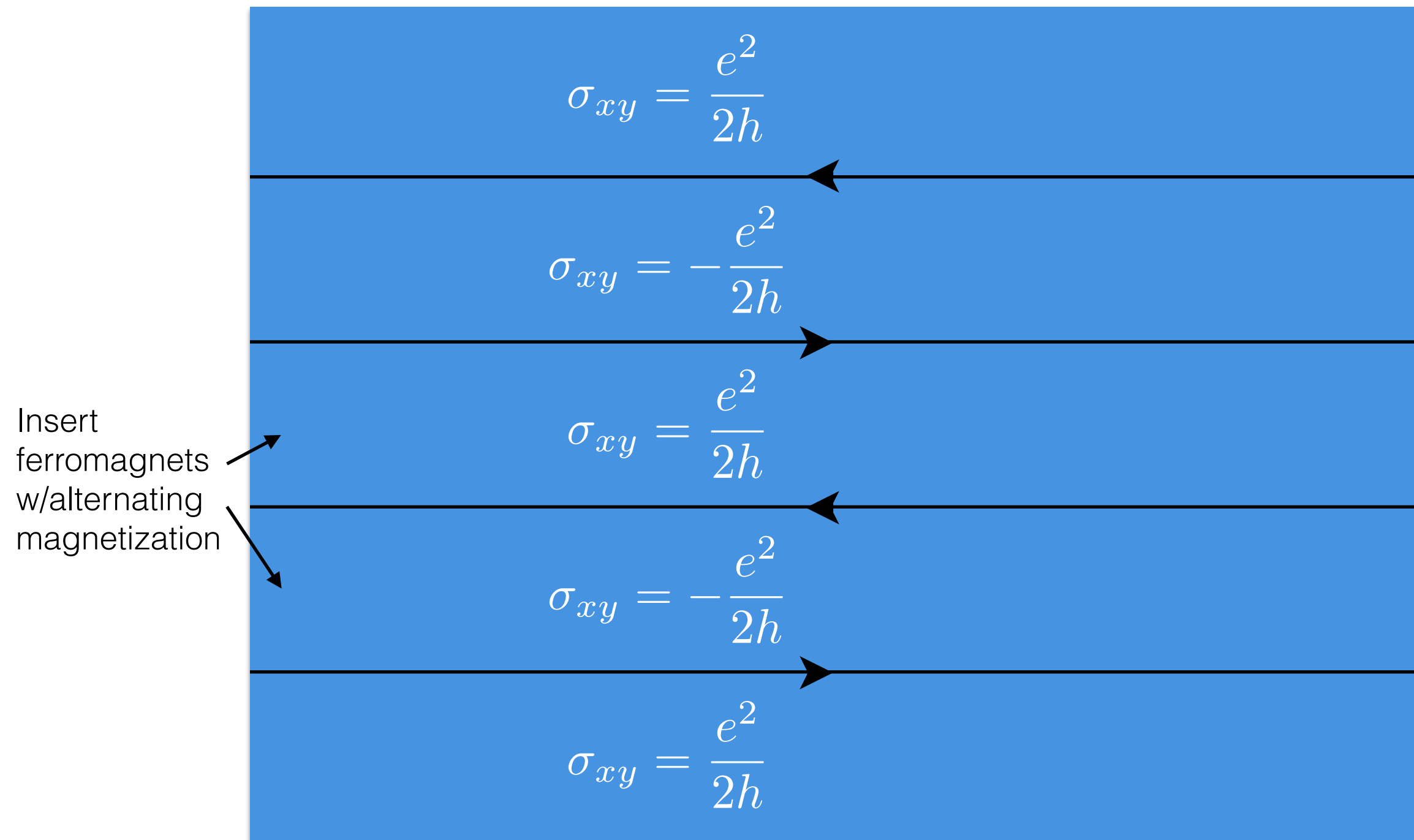
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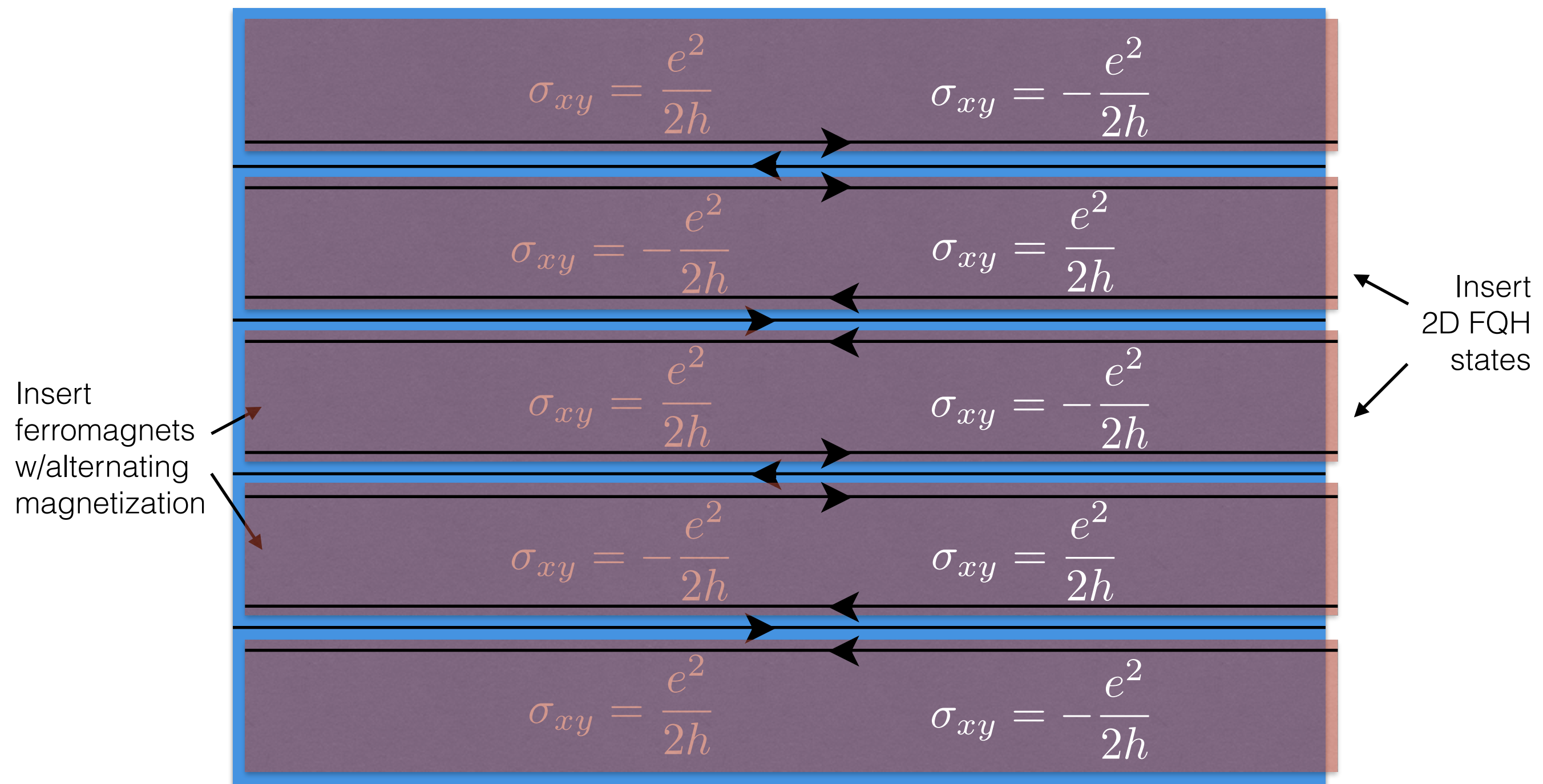
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Quasi-1D deformation allows rigorous analytical progress a la **Teo & Kane**.

# Accessing composite Dirac liquids

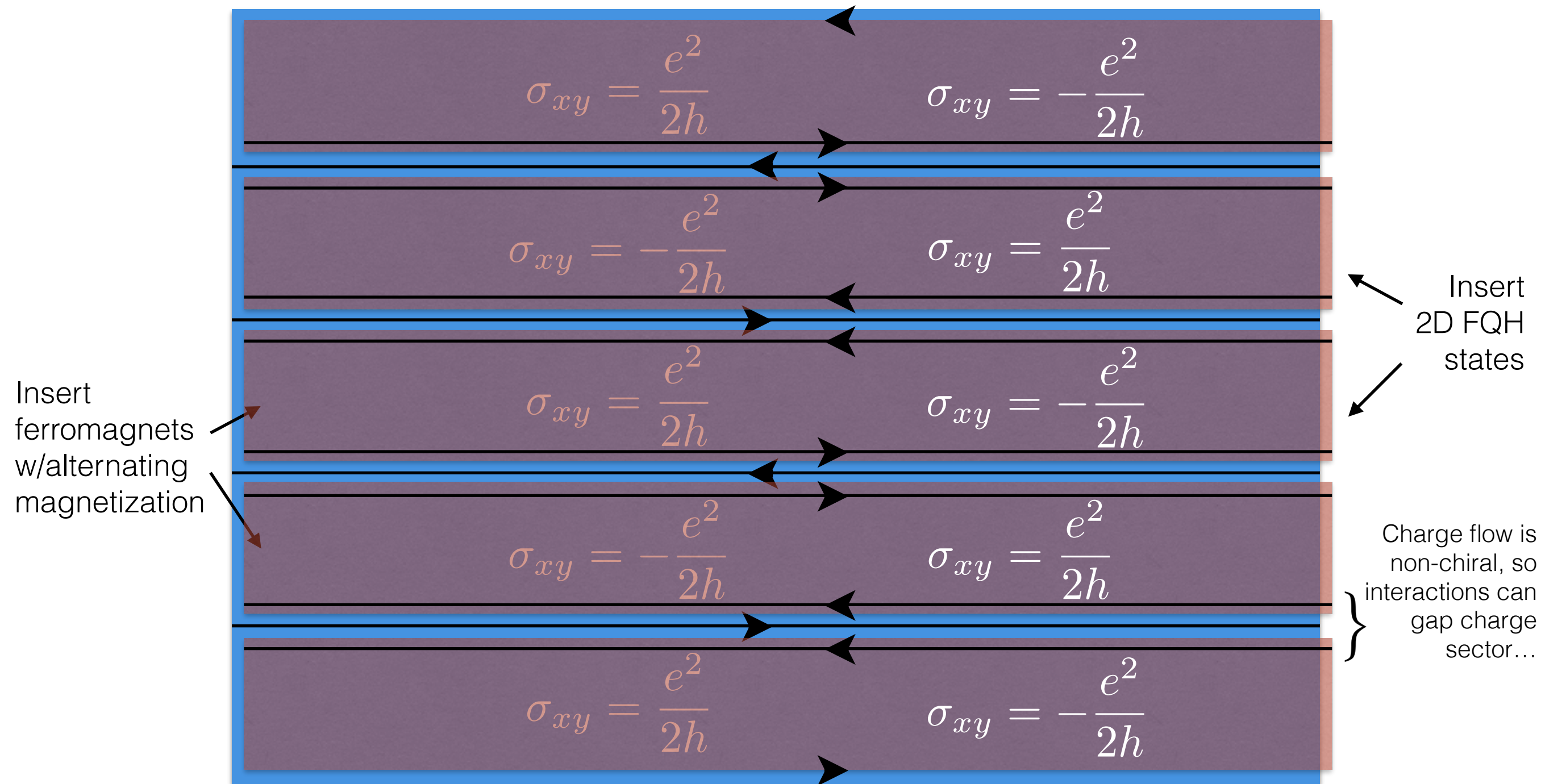
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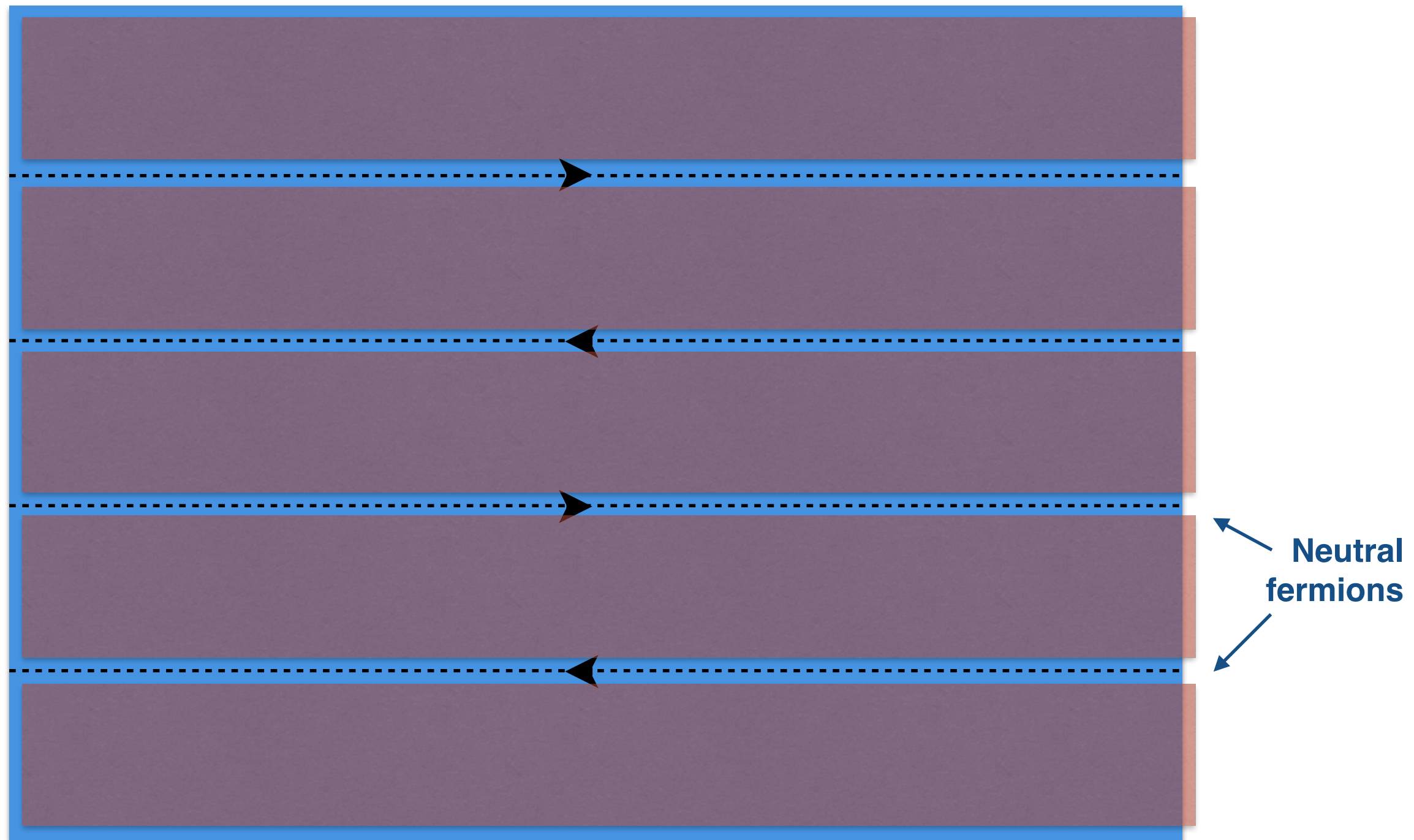
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**...but heat transport is chiral, so must have neutral modes left over!**

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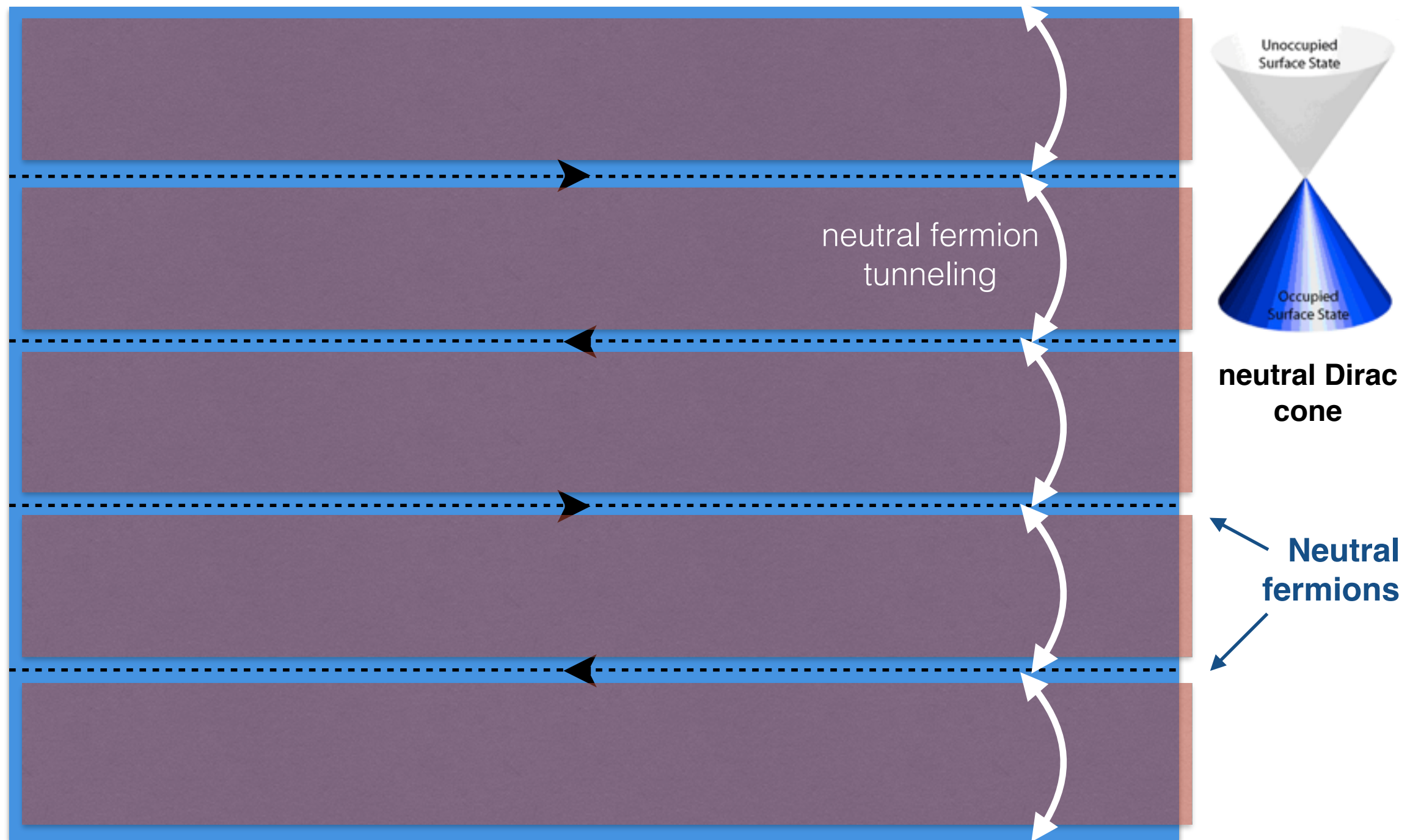
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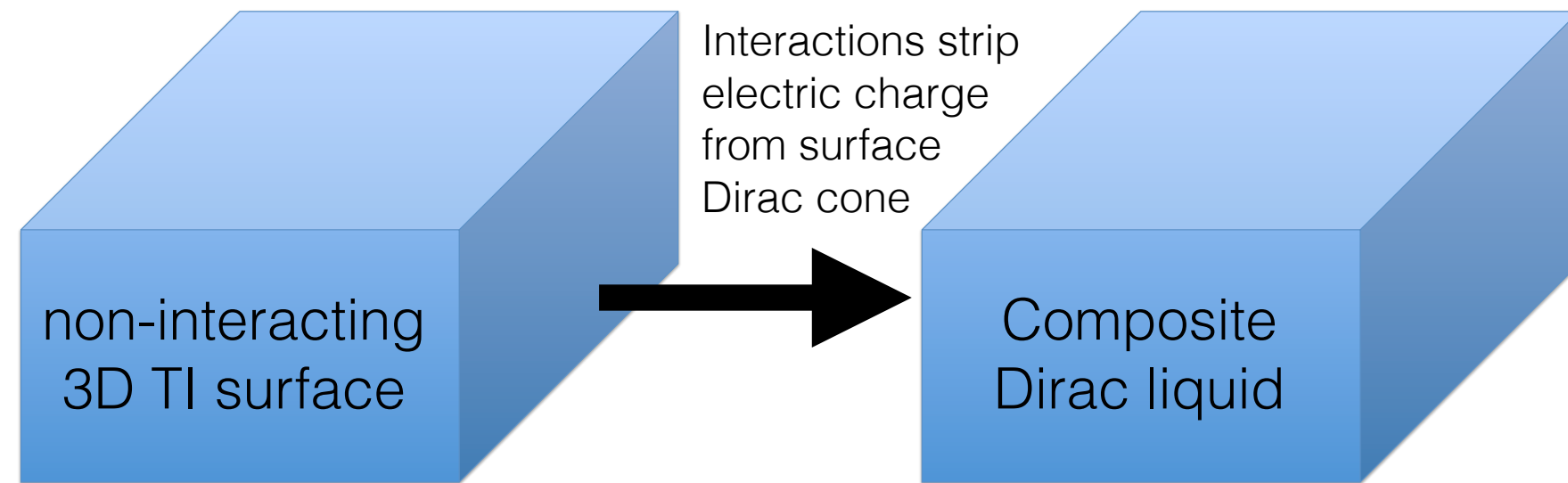
# Accessing composite Dirac liquids

## Composite Dirac liquid!

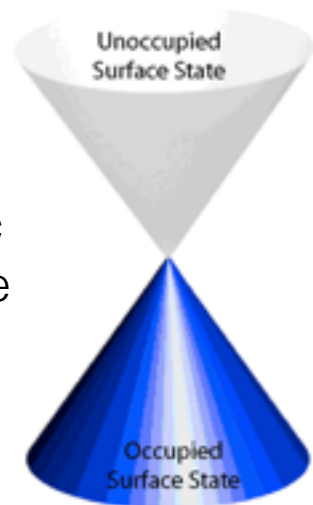


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# Nested Composite Dirac Liquids



**electronic**  
Dirac cone



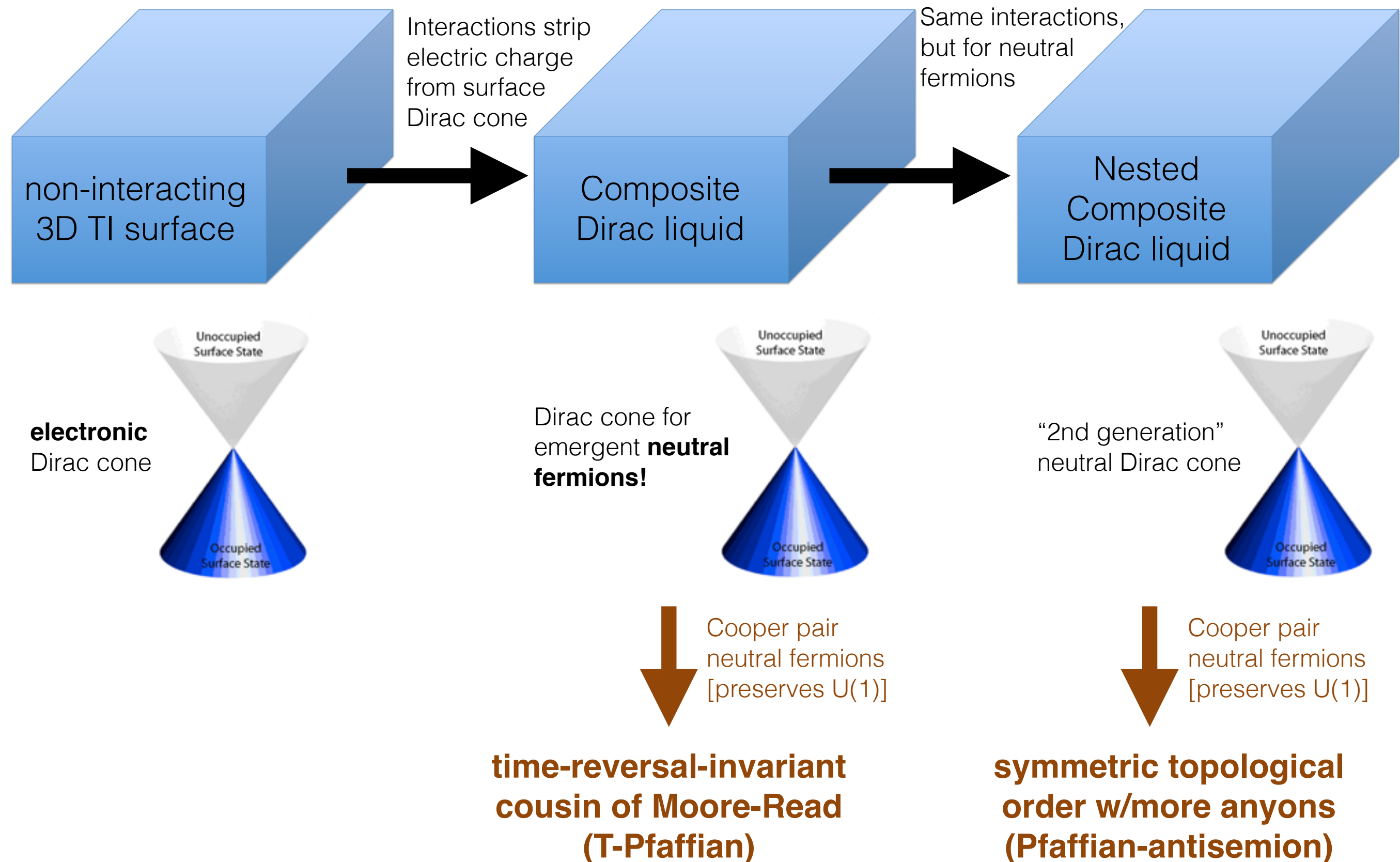
Dirac cone for  
emergent **neutral**  
**fermions!**



Cooper pair  
neutral fermions  
[preserves  $U(1)$ ]

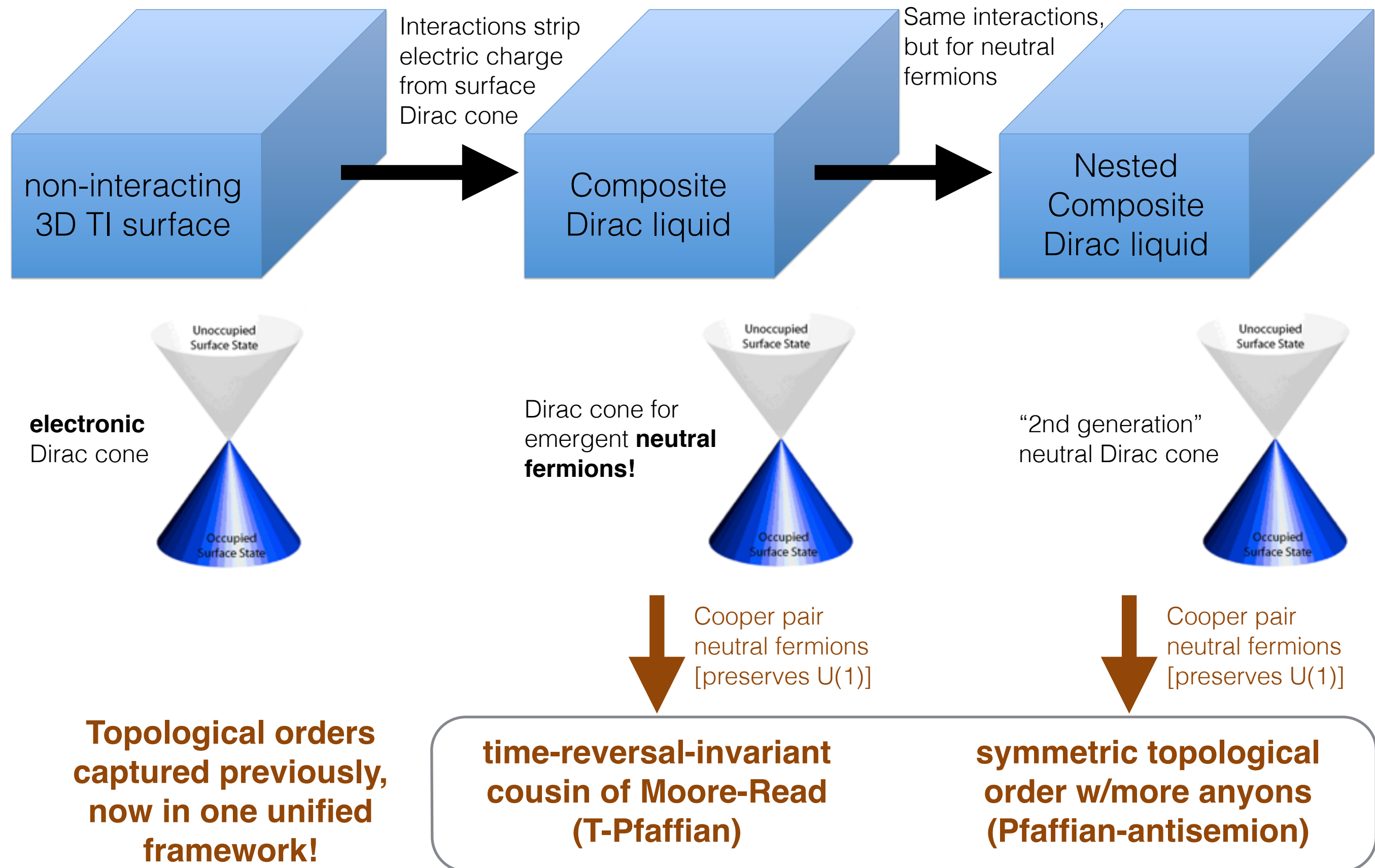
**time-reversal-invariant**  
**cousin of Moore-Read**  
**(T-Pfaffian)**

# Nested Composite Dirac Liquids



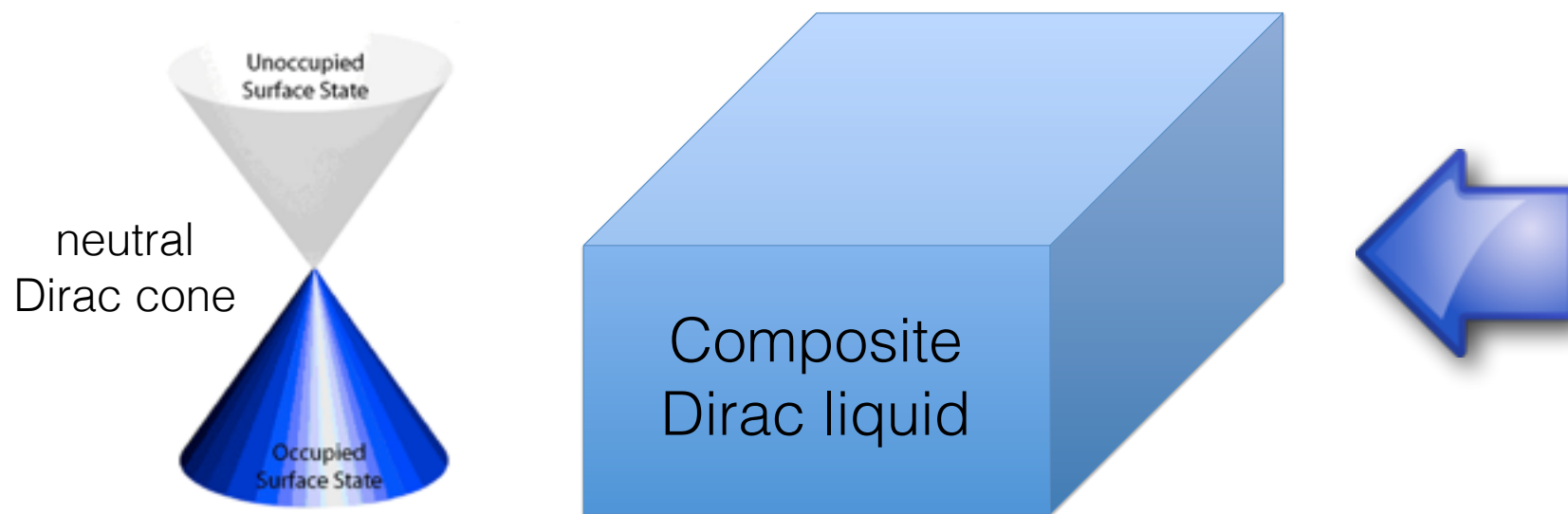


# Nested Composite Dirac Liquids



# Outlook

Interplay between interactions, symmetry-protected topological phases rich topic.



Provide unifying view on lots of physics, including quantum Hall; accessible via controlled analytical methods that deal with physical electrons.

Naturally extends to “weak topological insulators”.

(D. Mross, A. Essin, JA, A. Stern, in preparation)

Extension to 3D topological superconductors?

Isotropic implementations?

Quasi-realistic Hamiltonians for such states?

